

Volume-Enclosing Surface Extraction

B.R. Schlei*

GSI Helmholtz Centre for Heavy Ion Research GmbH, Planckstraße 1, 64291 Darmstadt, Germany

In this paper we present a new method, which allows for the construction of triangular isosurfaces from three-dimensional data sets, such as 3D image data and/or numerical simulation data that are based on regularly shaped, cubic lattices. This novel volume-enclosing surface extraction technique, which has been named VESTA, can produce up to six different results due to the nature of the discretized 3D space under consideration. VESTA is neither template-based nor it is necessarily required to operate on $2 \times 2 \times 2$ voxel cell neighborhoods only. The surface tiles are determined with a very fast and robust construction technique while potential ambiguities are detected and resolved. Here, we provide an in-depth comparison between VESTA and various versions of the well-known and very popular Marching Cubes algorithm for the very first time. In an application section, we demonstrate the extraction of VESTA isosurfaces for various data sets ranging from computer tomographic scan data to simulation data of relativistic hydrodynamic fireball expansions.

Keywords: image analysis, surfaces and interfaces, computed tomography, computational techniques; simulations, relativistic models, hydrodynamic models

PACS: 87.57.N-, 68.00.00, 87.57.Q-, 02.70.-c, 24.10.Jv, 24.10.Nz

I. INTRODUCTION

The determination of implicit surfaces, which are contained in three-dimensional (3D) image data and numerical 3D simulation data that are based on regularly shaped, cubic lattices, has become an important tool within many fields of science, industry and medicine (*cf.*, e.g., Ref.s [1, 2]). Such (usually triangular) surfaces can be used for visualization purposes [3], e.g., when 3D shapes that are contained in the data should be rendered, and/or they may represent the basis for further numerical evaluations. To this date, many articles have been written on the generation of triangular surfaces from 3D digital data sets (*cf.*, e.g., Ref.s [2, 4–14]).

A very popular algorithm for surface construction from 3D image data has been provided through the Marching Cubes algorithm (MCA), which has been developed by Lorensen and Cline in the mid-1980s [4]. Note, that this tool is nowadays still advertised (e.g., by the multinational NVIDIA Corporation [5]) as one of the state-of-the-art technologies for digital surface construction. Alternate approaches for 3D surface construction include – but are not limited to – the Marching Tetrahedrons [6, 7] (MTA), Marching Lines [8] (MLA; *cf.*, also Ref. [9]), the Cubical Marching Squares [10] (CMSA) algorithms, and VESTA [15, 16], which is described in this paper.

The MCA is a template-based approach, and as a consequence of the non-trivial topology in 3D [11], several surface templates have been initially overlooked, resulting in the accidental generation of holes in some data sets [2]. Meanwhile, this problem has been fixed (*cf.*, e.g., Ref.s [12, 13]). For a recent and very detailed discussion on the MCA’s history, *cf.*, Ref. [14]. The MTA, unfortu-

nately, has directional ambiguities, because it subdivides a cube with tetrahedrons, which cannot be done symmetrically [6, 7]. The MLA – and the similar techniques – are not template based [8, 9], but they require further processing, so that apparent cell ambiguities are properly resolved (*cf.*, e.g., Ref. [11]). The CMSA is very similar to the previous, cubically based approaches, however templates are used for the faces of the $2 \times 2 \times 2$ voxel neighborhoods under consideration, and further work to resolve cell ambiguities is done as well [10].

The “Volume-Enclosing Surface exTraction Algorithm” (VESTA), which is presented in this paper, allows one to numerically compute – very fast and totally robust (i.e., without the accidental generation of any holes) – non-degenerate, mathematically well oriented triangular surfaces in 3D [15]. VESTA constructs surfaces – so to speak – from the grounds up, because it continuously transforms the initial surface that consists of the boundary faces (squares) of selected voxels into the final isosurface. Instead of using many different surface templates, VESTA uses a single building block that is based on the vector decomposition of a single voxel face (*cf.*, Fig. 2.b). VESTA collects all of the participating vectors and groups them into closed vector cycles, while resolving 3D cell ambiguities properly. In fact, this is done in analogy to the DICONEX algorithm [17], which allows one to construct gap-free contours for 2D digital data.

The original VESTA algorithm is not limited to the processing of $2 \times 2 \times 2$ voxel cell neighborhoods, and it therefore minimizes the generation of redundant information. Since VESTA surfaces can be generated in a global “disconnect”, a global “connect”, and a “mixed” connectivity mode, and since all of these three modes can be executed either in a low resolution (“L”) or in a high resolution (“H”) mode, VESTA can produce in total six different types of surfaces on demand. Since all VESTA surface cycles are confined to $2 \times 2 \times 2$ voxel cell neighbor-

*Email address: schlei@me.com

hoods at all times (*cf.*, below), a marching 3-cell variant is presented in this paper as well. The marching VESTA makes use of a very simple, but explicit table (*cf.*, Table 1, subsection II.F.) of directed vector paths for the construction of closed vector cycles, and it is therefore very easy to implement into computer code.

This paper is organized as follows. First, we shortly review the DICONEX algorithm [17], which uses a 2D digital data set as input, because VESTA can be viewed as an extension of DICONEX from 2D into 3D. Then we shall explain VESTA in great detail. In particular, we shall compare VESTA with both: (i) the original MCA [4], which can be found in many text books (*cf.*, Ref.s [2, 5]), but which may generate holes; and (ii) a state-of-the-art MCA implementation [18] by Bourke et al., which represents an extended version of the original MCA that does *not* generate any holes and that produces exactly the same surface as VESTA, when it is executed in its low resolution (“L”), global “disconnect” mode. The in-depth comparison with the original MCA is done within the theoretical section, whereas the comparison with the extended MCA is done within three VESTA application related subsections. Finally, this paper will conclude with a short summary.

II. THE SURFACE EXTRACTION FRAMEWORK

Before we explain the surface extraction with VESTA, we first consider contour extraction in 2D with DICONEX.

A. DICONEX - Dilated Contour Extraction

In Fig. 1.a, we show a binary image with $6 \times 6 = 36$ pixels (i.e., picture elements). Let us assume, that the gray pixels have been segmented, i.e., they have been selected for an enclosure. In the following, we shall denote segmented pixels as “active” and the other pixels as “inactive” pixels. The objective is now to enclose the 11 active (gray) pixels with contours while making use of the DICONEX algorithm [17]. First all initial contour vectors (ICV) ought to be collected. The ICVs are here oriented pixel edges that separate an active pixel from an inactive one.

E.g., for a single active pixel that has no active next neighbor at all (*cf.*, Fig. 1.b), we have a maximum of four single ICVs (in the figure, they are numbered from 0 to 3). Note, that an active pixel lies always to the left of its corresponding ICV. Because of this construction, it is possible to forward the information of the interior and/or exterior of a shape (i.e., a collection of segmented pixels) that needs to be enclosed by a finite number of contours. Note that a shape is enclosed counterclockwise, while its potential holes are enclosed clockwise.

In a second step, the ICVs will be connected to oriented shape-enclosing contours. Note, that each ICV

both starts and ends in a single juncture (*cf.*, Fig. 1.c). Most of the times it is trivial to find the successor of an ICV, if we attempt to determine the particular IVC connectivities. But sometimes it is possible that a single active pixel is in contact with another active pixel through only one single point (*cf.*, white dot in Fig. 1.c). The latter results in a situation where we have two incoming and two outgoing ICVs for this particular point of contact (*cf.*, Figs. 1.c and 1.d). In the following, we shall call such a juncture “point of ambiguity” (POA). In order to avoid gaps in the final set of contours, one has to treat these POAs in a special way (*cf.*, Ref. [17]).

In Fig. 1.d, a connectivity diagram is depicted. If one connects to an incoming IVC an outgoing IVC that belongs to the same active pixel as for the incoming IVC, then one performs a left turn, i.e., one follows one of the paths of the white bent vectors. Conversely, if one connects to an incoming IVC an outgoing IVC that belongs to another active pixel as for the incoming IVC, then one performs a right turn, i.e., one follows one of the paths of the black bent vectors instead. The particular turns will either lead to a separation (“disconnect” mode), or to a joining (“connect” mode) of shapes next to a POA, respectively.

A user can interactively make this consistent choice of

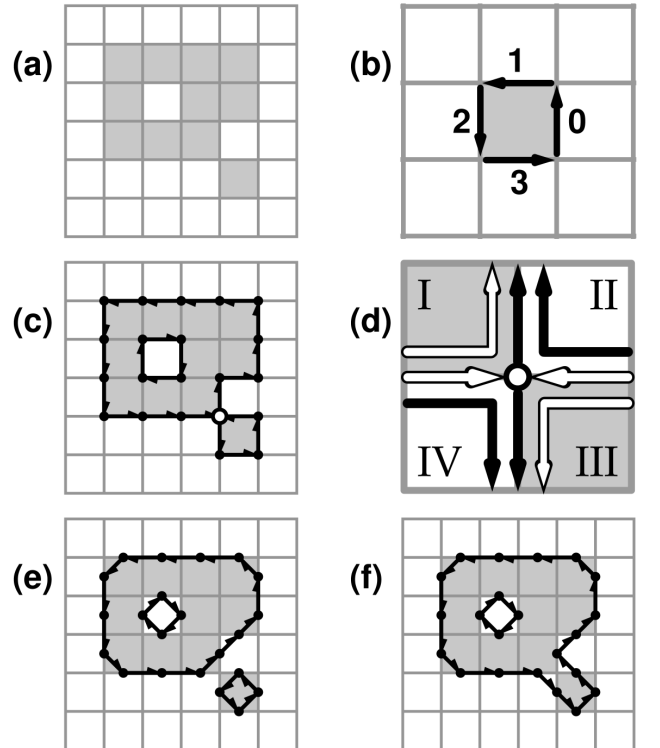


FIG. 1: (a) Binary image; (b) initial contour vectors for a single pixel; (c) initial contour vectors and junctures (black dots) for the binary image; (d) connection diagram for a single point of ambiguity (white dot); (e) dilated contour vectors and support points (black dots) resulting from the “disconnect” mode; (f) as in (e), but resulting from the “connect” mode.

either left or right turns within one given image, e.g., if only binary information is available, in order to ensure the construction of *gap-free* contours. Note that this rather global decision making process may be replaced and automated by a local decision on the turning of the IVCs, while using, e.g., the average gray level of the four pixels, which surround a particular POA, provided that gray level information is available. In Figs 1.e and 1.f, the final DICONEX contours are depicted for the “disconnect” mode and for the “connect” mode, respectively. These contours have been obtained from a displacement of the origins of the IVCs to the middle points of their corresponding pixel edges.

In essence, the DICONEX contours result from a displacement of the initial pixel edges that separate active pixels from inactive ones. Note that the set of these initial pixel edges already provides a perfect enclosure of the segmented pixels. In 3D, we shall proceed now analogously.

B. Initial Considerations for Voxels

VESTA will enclose voxels that have an inherent field value, e.g., a shade of gray, above an initially given threshold. A voxel (i.e., VOLUME piXEL) is a 3D object. To be more specific, it is represented by a cube and, hence, it has six squares as faces. For an active voxel, i.e., a voxel that should be enclosed by a surface section, VESTA will check all of its six nearest neighbors. If there is a transition from an active to an inactive voxel, the corresponding voxel face will be recorded.

In Fig. 2.a, two neighboring voxels are shown. One voxel, i.e., the active voxel, is marked with a sphere at its center. The second voxel has no sphere at its center, because it is considered inactive, i.e., this voxel has a field value below the initially given threshold. Between an active and an inactive voxel we shall have a contribution to the enclosing surface. Therefore, the face that separates these two voxels has to be considered.

In Fig. 2.b, we show such a boundary face. The center of this face is marked with a black dot. Such voxel face centers will finally be support points of the VESTA surface. In case an isosurface should be constructed, the voxel face centers may be moved within the bounds of its corresponding range vector, r , which has its origin at the center of the active voxel and which ends in the center of the neighboring inactive voxel. Fig. 2.c helps to illustrate, how VESTA surfaces can be transformed into isosurfaces while considering the 2D analog of two neighboring pixels.

Two pixels - one with a gray-level value of 133, the other one with a zero valued gray-level - are initially separated by a contour section that is located exactly in the middle between them (dotted line). A range vector connects the centers of the two pixels. The centers of each pixel are assumed to correspond exactly with their gray-level values. Since the isocontour is supposed

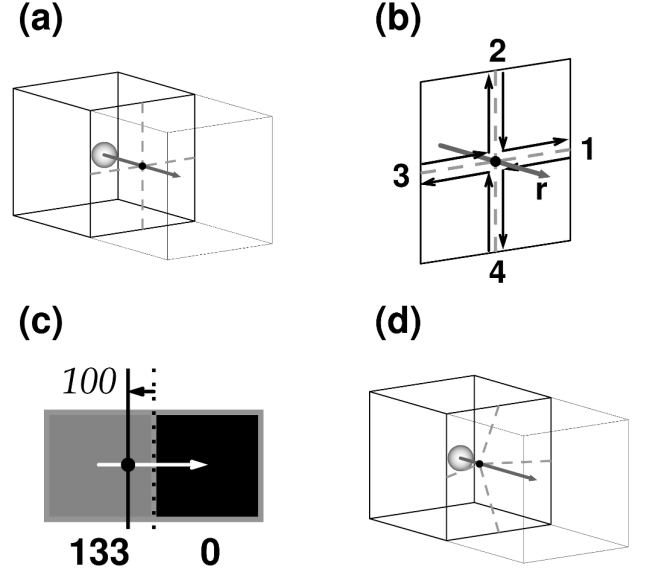


FIG. 2: (a) Two voxels (one is active (sphere) and one is inactive), which are in contact through a voxel face, and a corresponding range vector; (b) a voxel face with its voxel face center (black dot), its range vector, r , (gray), four edge middle points (1, 2, 3, 4, which each are connected with a dashed line to the voxel face center), and voxel face vectors (black); (c) contour displacement for a pixel pair (see text); (d) as in (a), but with a displaced voxel face center.

here to represent a gray-level of value 100, it should not be positioned at the middle of the range vector. This medium position actually represents a gray-level of value 66.5 while assuming a linear interpolation between the gray-level bounds. In fact, the “true” location of the support point for the isocontour is located closer - and therefore has to be shifted - towards the center of the pixel with the gray-level of value 133. Hence, the isocontour (solid line) is supported by a point, which is located within the pixel with the gray-level of value 133.

In Fig. 2.d, we show the dislocation of the boundary face center in 3D due to the previous outline for 2D[36]. Note that the dashed gray lines each continue to provide a connection from the boundary face center to one of the four edge middle points of the boundary face. In the following, these four edge middle points, which are numbered counterclockwise from 1 to 4 in Fig. 2.b (i.e., if we apply a right-hand-rule to the perpendicular range vector), will assume the role of junctures (see below). The following ansatz will help to provide a connectivity among the voxel face vectors (VFFV) within a given boundary face. Let us unite the eight black VFFVs of Fig. 2.b into four vector pairs as follows: connect the juncture 4 (3, 2, 1) via the face center to the juncture 3 (2, 1, 4). As a consequence, one obtains for each boundary voxel face the four internal paths $4 \rightarrow 3$, $3 \rightarrow 2$, $2 \rightarrow 1$, and $1 \rightarrow 4$, respectively.

Any single isolated active voxel or any active voxel cluster will initially be fully enclosed by a certain number

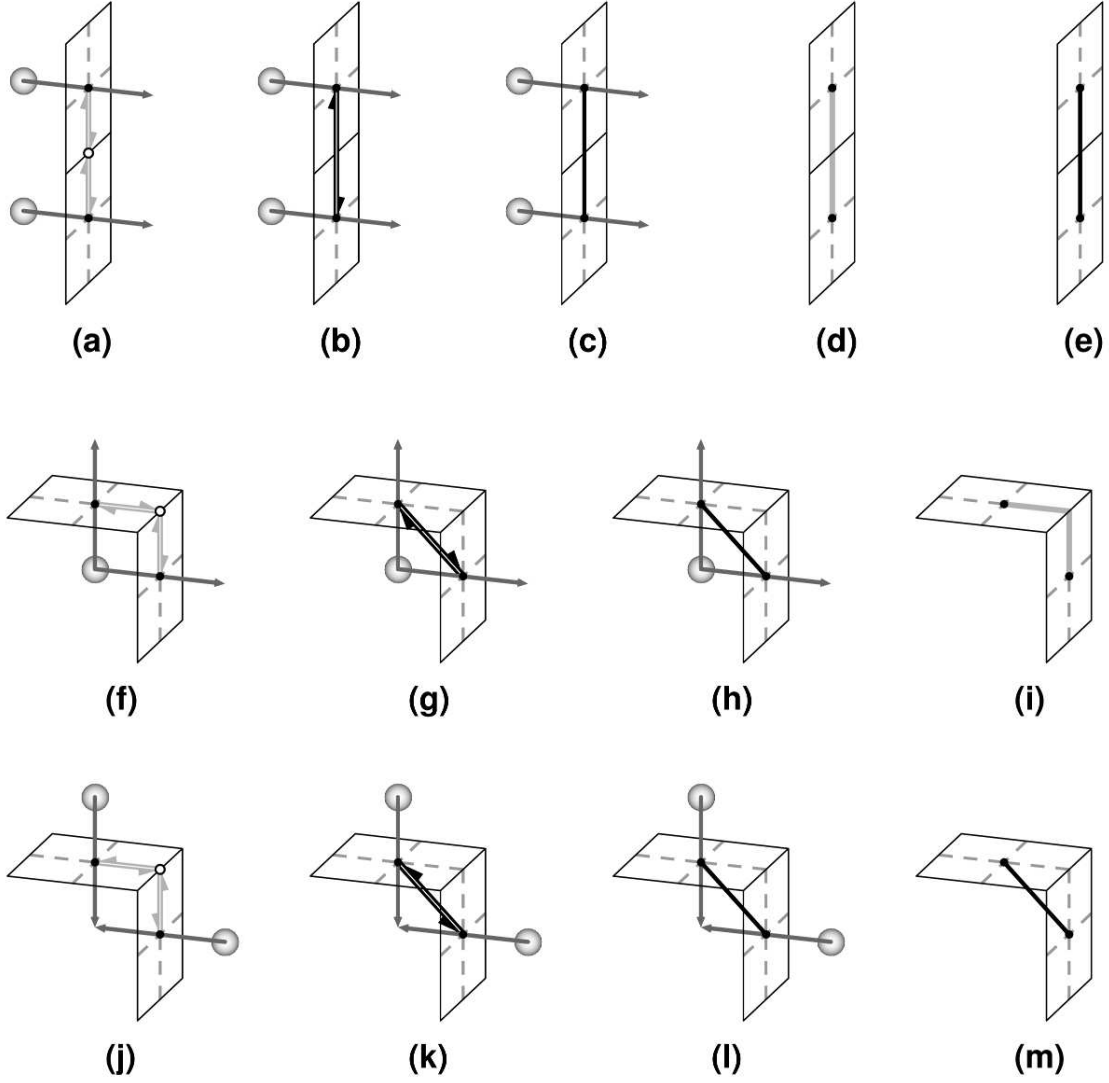


FIG. 3: Pairs of boundary faces, which are in direct contact (see text); (a) – (e) in-plane, for two active voxels each; (f) – (m) out-of-plane, for either one or three active voxels each; the spheres indicate active voxels, which are shown only for corresponding boundary faces, and in conjunction with their corresponding range vectors; light gray vector pairs and thick light gray lines represent voxel face vectors, whereas black vector pairs and thick black lines represent VESTA cycle vectors; black dots are VESTA surface support points, whereas white dots represent junctures.

of boundary faces. Hence, each boundary face will be in contact with at least another boundary face through one of its four edges or – to be more specific – through one of its four junctures. Without loss of generality, we shall discuss in the following how pairs of boundary faces may connect in 3D. In Fig. 3, the various possible configurations are shown. Through each juncture (white dots) two VFV pairs may be connected (*cf.*, Figs 3.a, 3.f, and 3.j). While doing so, it is only permitted to attach the origin of a given VFV to another ones tip. Hence, these vector pairs will yield oriented paths from one boundary face center to another one.

If we ignore the junctures, the newly formed vector pairs can be replaced by single vectors, which each will connect one VESTA surface support point with another

one (*cf.*, Figs 3.b, 3.g, and 3.k, respectively). In the following, the latter vectors will be called “VESTA cycle vectors”. Throughout this paper, vector pairs of antiparallel vectors will sometimes simply be drawn as single lines (*cf.*, Figs 3.c, 3.d, 3.e, 3.h, 3.i, 3.l, and 3.m). Furthermore, it may not be specified in one of the following drawings, on which side of a given boundary face the active voxel resides. Then again, single vectors may be drawn as well as single line segments where no particular orientation will be indicated, since it should be obvious from the particular context.

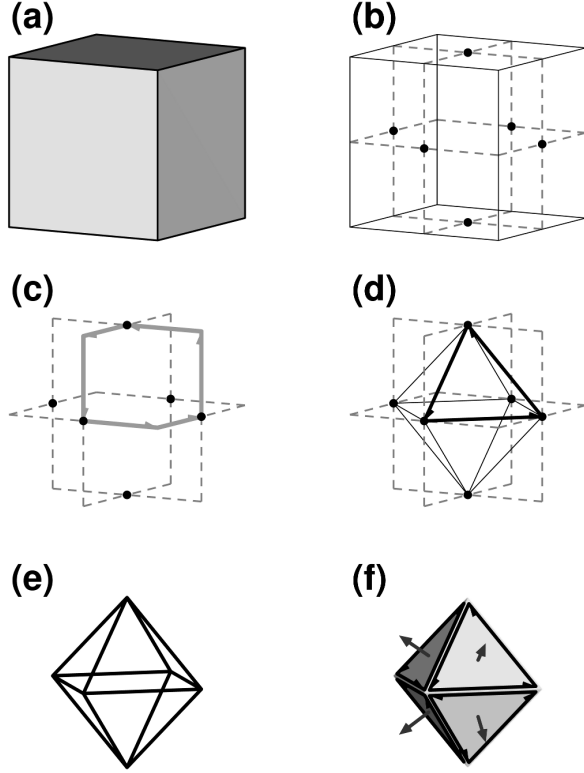


FIG. 4: (a) A single voxel (gray cube); (b) the six boundary faces of the single voxel (*cf.*, Fig. 2.b); (c) six voxel face vectors form a single vector cycle; (d) replacement of voxel face vector pairs with VESTA cycle vectors (*cf.*, Figs 3.f and 3.g); (e) resulting wire frame after the processing of all voxel face vectors; (f) an octahedron, the corresponding final VESTA surface, which is superimposed with oriented 3-cycles and normal vectors for the visible gray triangles.

C. VESTA - a Volume-Enclosing Surface extraction Algorithm

In the previous subsection, we have discussed the basic processing steps that will allow for a proper surface extraction. We shall begin with the processing of a single active, isolated voxel (*cf.*, Fig. 4.a). In Fig. 4.b, this voxel is represented by its six boundary faces. As an example, only for three of the boundary faces, and only for a single quadrant of each of these three faces, those VFVs are shown in Fig. 4.c, which form a closed vector cycle. On the one hand, Fig. 2.b has made clear that VFVs are pairwise connected by the boundary face centers; on the other hand, Fig. 3.f suggests, that VFVs can be pairwise connected through the junctures.

After the replacement of VFV pairs that are connected through junctures by the proper VESTA cycle vectors (*cf.*, Fig. 3.g), one obtains in Fig. 4.d a closed VESTA 3-cycle, which represents a single oriented triangle. The processing of all VFVs yields seven further triangles. As a result, one obtains a fully closed and oriented VESTA surface (*cf.*, the octahedrons in Figs 4.e

and 4.f). In Fig. 4.f, the octahedron is superimposed with the four normal vectors of the four visible surface triangles. These normal vectors point to the exterior of the enclosed shape.

All initially given 48 VFVs are unique. In total, one obtains after their processing 24 VESTA cycle vectors for the 12 edges of the octahedron, i.e., each edge represents two antiparallel vectors. Note, that VESTA will always reproduce 2D DICONEX contours whenever a final 3D VESTA surface is intersected with a principal plane at the corresponding centers of the active voxels (*cf.*, Figs 31.a – 31.c).

In general, one has to process more complex shapes than just single voxels. Let us assume for the remainder of this subsection that the 3D data set is simply binarized (as in “active” and “inactive” voxels). The more general situation will be discussed in one of the next subsections. As in 2D, we shall encounter 3D junctures in the process of VESTA surface cycle formation that will play the role of POAs. In Fig. 5, we show all possible nine configurations of boundary faces that can be in direct contact. Gray solid lines represent single VFVs in the figure. Most of the configurations show paths, which do not self-intersect. However, in Figs 5.e and 5.g, we observe self-intersections, which are due to the fact that two voxels are in contact with one another through only one single edge (*cf.*, Fig. 6.a).

Fig. 6.a also shows a dotted square, which has the common juncture of the four boundary faces that are in direct contact at its center. As an example, one incoming and two outgoing VFVs are shown for this juncture. The upper left connection diagram of Fig. 6.b shows this configuration once again. However, due to the 3D nature of the problem we actually have four incoming and four outgoing VFVs at this particular juncture. Clearly, this juncture is a POA, and Fig. 6.b shows for each of the four incoming VFVs a connection diagram with two valid outgoing VFVs. In order to avoid holes in the final set of surface tiles, one has now uniformly to choose among the two following possibilities for all four connection diagrams.

Either, one selects as successor to a given VFV the one that is connected with its origin and that belongs to the same voxel; then one generates the “disconnect” mode while following the paths of the white bent vectors (*cf.*, Fig. 6.c). Or, one selects as a successor the one that is connected with its origin and that belongs to the other voxel; then one generates the “connect” mode while following the paths of the black bent vectors (*cf.*, Fig. 6.d). This selection process is the 3D analog of the 2D selection process (*cf.*, Fig. 1.d). A consistent selection of these successors is the key step within VESTA, which prevents a *tearing of holes* into the final surface. The user has to perform a global selection of one of the two modes, if only binary data are available. In Figs 6.e and 6.f, the final VESTA surface wire frames are depicted for the “disconnect” mode and for the “connect” mode, respectively. Note, that each edge of the wire frames consists

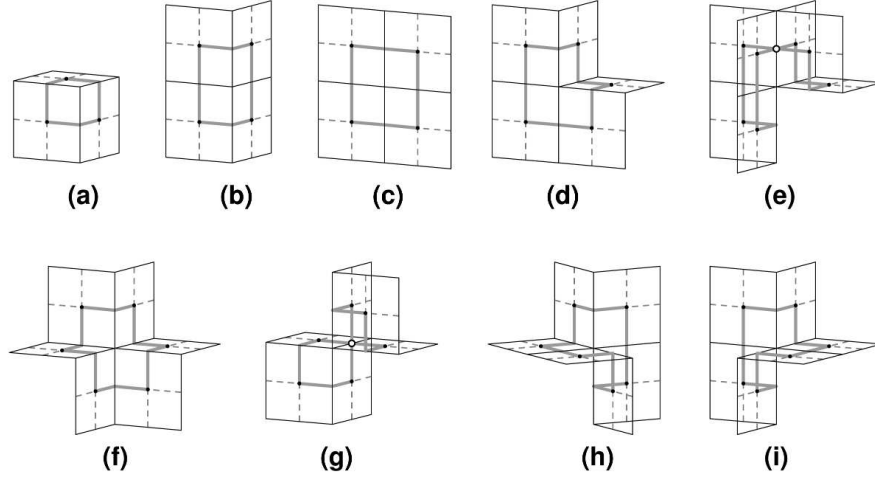


FIG. 5: Nine boundary face chains with their corresponding voxel face vectors; note, that the directions of the vectors have been omitted, because it is not specified on which side of the boundary faces the active voxels reside; in (e) and (g), the white dots mark junctures that are points of ambiguity.

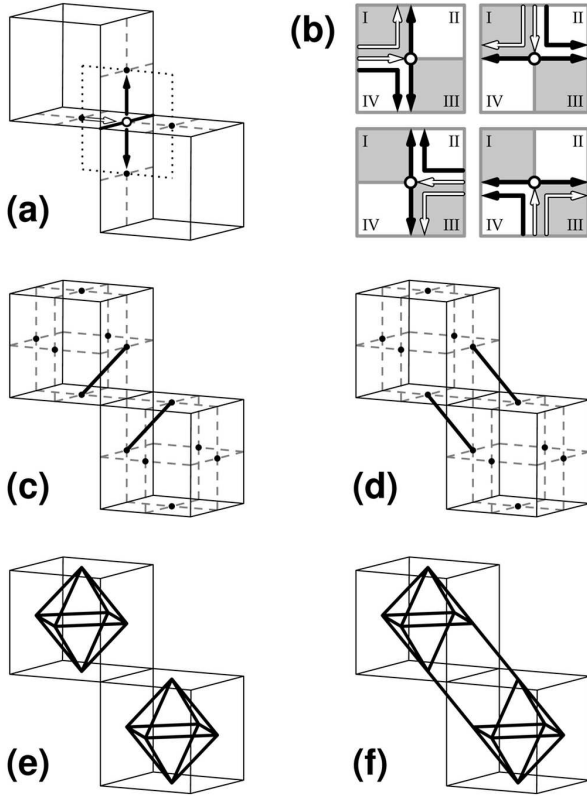


FIG. 6: Two voxels in contact through one single voxel edge (a) the dotted square contains a single point of ambiguity (white dot) with one incoming (white) and two outgoing (black) voxel face vectors; (b) four connection diagrams for a single point of ambiguity (white dot); (c) with VESTA cycle edges for the “disconnect” mode, (d) with VESTA cycle edges for the “connect” mode; final wire frames for (e) the surfaces in “disconnect” mode, and (f) the surface in “connect” mode.

of two antiparallel VESTA surface cycle vectors.

In Fig. 7, all possible nine VESTA surface cycles are shown (*cf.*, Fig. 5), which can be generated while uniformly either choosing the “disconnect” or the “connect” mode for the whole 3D data set under consideration. Note, that these planar and nonplanar cycles can be traversed both ways, depending on the orientation of the range vectors (not shown here) of the contributing active voxels. In particular, the cycles in Figs 7.e and 7.g are the VESTA surface cycles of maximum length; their separated counterparts can be seen in Figs 13.b and 13.a, respectively. Furthermore, and as an important result, all of the cycles are confined to a $2 \times 2 \times 2$ voxel neighborhood at all times. They are supported alone by the initial boundary face centers, which may vary – if necessary – within the bounds defined by their corresponding range vectors (*cf.*, Fig. 2). Finally, it should be noted that more than one VESTA surface cycle can appear within a $2 \times 2 \times 2$ voxel neighborhood (*cf.*, Fig. 13).

D. VESTA Surface Cycle Decomposition

The VESTA surface cycles, which have been introduced in the previous subsection, have to be processed further, if one wants to obtain surfaces, which are decomposed into triangles. In fact, the only cycle that requires no further processing is the one that is shown in Fig. 7.a. If we demand to use no further points while inserting edges, we may end up with a result that is shown in Fig. 8. Note that instead of inserting single edges, one rather has to insert antiparallel vector pairs for the proper breakup of the N -cycles ($N > 3$) into 3-cycles. This allows one to pass on the initial orientation of the N -cycles to the newly formed 3-cycles.

In Fig. 8, the triangular surface tiles have been chosen similar to the template tiling, which has been proposed

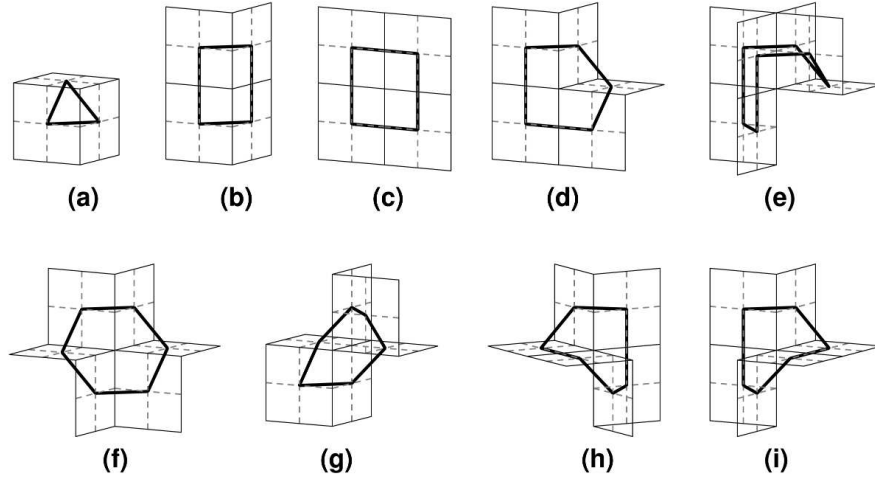


FIG. 7: Nine VESTA surface cycles with corresponding boundary face chains: (a) (planar) 3-cycle, (b) & (c) planar 4-cycles, (d) nonplanar 5-cycle, (e) nonplanar 7-cycle, (f) planar 6-cycle, (g) – (i) nonplanar 6-cycles.

by various authors of the MCA (*cf.*, Ref.s [4] and [18]). However, this tiling is not unique, since other subdivisions are possible. If no further points are used for the decomposition of VESTA N -surface with $N > 3$, biases may be introduced with respect to the convexity and/or concavity of the local surface sections (*cf.*, e.g., Fig.s 18.e and 18.f). In cases, when more numerical accuracy is desired, it may be advisable to introduce for each N -cycle with $N > 3$ an additional point that lies within the cycle (*cf.*, e.g., Fig.s 18.c and 18.d).

In Fig. 9, the N -cycles ($N > 3$) of Fig. 7 have been broken down into 3-cycles (i.e., oriented triangles) while using as additional point the average of all involved cycle support points. Note that in the case where the initial boundary face centers are moved within the bounds of the range vectors, the average point should be determined after this movement in order to save computing time. Admittedly, this choice of the average is a simple one, but it will be used for the remainder of this paper. Ultimately, it is left to the designer of a particular VESTA implementation, whether a more elaborated approach should

be used for the breakup of the surface cycles.

E. More on Voxel Connectivity

In the case, that two neighboring active voxels only share a single point (*cf.*, Fig. 10.a), VESTA will not connect the two volumes, since none of the involved VFVs of one voxel will ever meet one of the VFVs of the other voxel. In Fig. 10.b, we show the wire frames (i.e., two octahedrons) for this configuration. Apparently, this connection is in the discretized 3D space too weak that it should matter. If one – nonetheless – desires to establish a link between voxels that only share a single common point one may, e.g., insert a tubular template (while removing the two opposite triangles, i.e., 3-cycles, at the same time) as indicated through the six dashed lines in the figure. However, we shall not use such an approach here, since VESTA by itself does not establish this kind of connectivity.

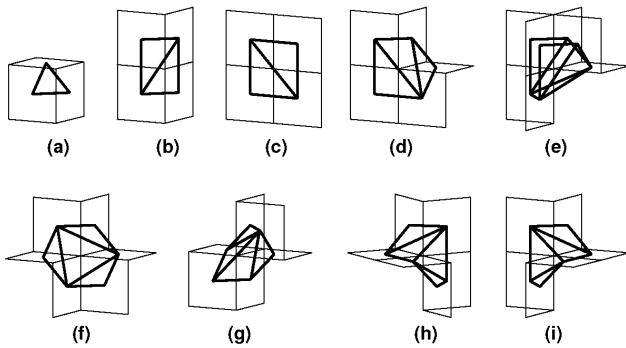


FIG. 8: Nine VESTA surface cycles with decomposition into triangles: (a) 3-cycle, (b) & (c) 4-cycles, (d) 5-cycle, (e) 7-cycle, (f) – (i) 6-cycles. No additional surface support points are used.

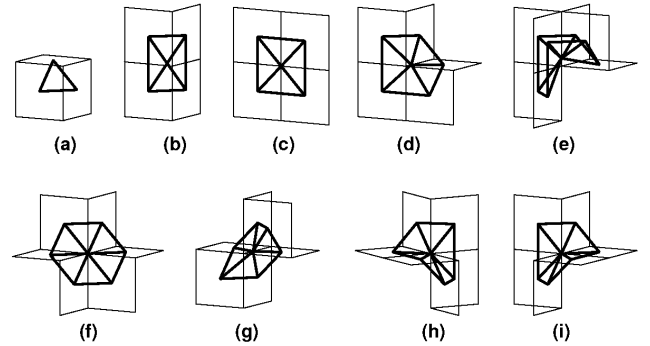


FIG. 9: Nine VESTA surface cycles with decomposition into triangles: (a) 3-cycle, (b) & (c) 4-cycles, (d) 5-cycle, (e) 7-cycle, (f) – (i) 6-cycles. For cycles with $N > 3$, average points are used to generate a surface tiling with triangles.

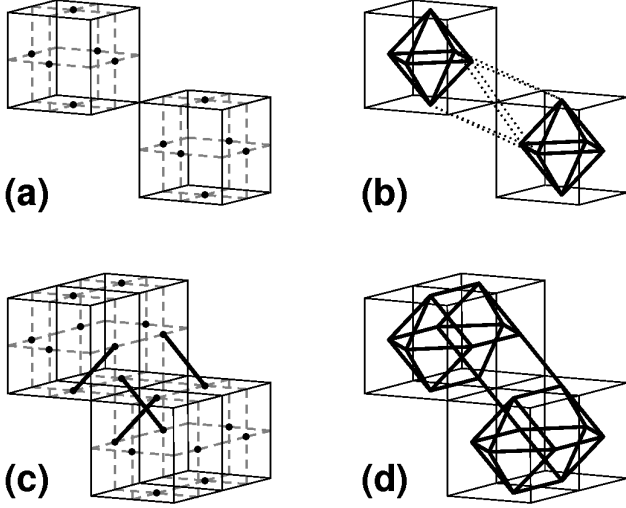


FIG. 10: (a) Two active voxels that have only a single point in common; (b) the VESTA wire frame for the voxels shown in (a), together with a connecting triangular tube template (dotted lines); (c) two active voxel pairs in direct contact, which each just have a single edge in common, but with different connectivity modes; (d) the wire frame for the voxels shown as in (c).

So far, we have discussed only the uniform and global usage of either the “disconnect” or the “connect” mode for the processing of the 3D data set under consideration. However, if more than just plain binary information (such as “do enclose” and “do not enclose a voxel with a surface”) is contained in the data, e.g., gray-level information, we can *locally* define through a threshold, which mode should be (consistently) applied. Ultimately, the interaction of the user is no longer required, since the thresholding could now be applied through automation.

In Fig. 10.c, we show two pairs of active voxels, which each just share a single edge, and that are in direct contact to each other. Here, we consider the case that for each of the two neighboring pairs a different connectivity mode has been selected as it is indicated in the figure. As a consequence of the tracing of VFVs (*cf.*, Figs. 11.a), a new VESTA surface cycle comes into existence. This 8-cycle is shown in Fig 12.a. The corresponding wire frame of the two voxel pairs is depicted in Fig 10.d where each edge of the wire frame consists of two antiparallel VESTA surface cycle vectors. Note, that if both connection modes would have been the same, e.g., “connect mode”, we would have obtained the two VESTA 4-cycles as shown in Fig. 13.c. In Fig. 14.a, the newly formed 8-cycle is decomposed into 3-cycles while using an additional average point of the involved cycle support points.

In fact, many more VESTA surface cycles can appear, dependent on the internal features of the considered 3D data. All of these are shown in Fig. 12; and in Fig. 14, we show their decomposition into triangles while using average points. In contrast to VESTA, the original MCA [4] *does not* propose templates for the here

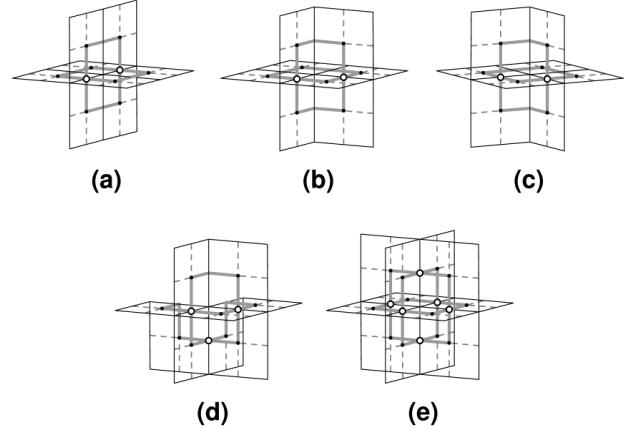


FIG. 11: Four further boundary face chains with their corresponding voxel face vectors; the directions of the vectors have been omitted, because it is not specified on which side of the boundary faces the active voxels reside; the white dots represent points of ambiguity; note, that figures (b) and (c) are equal, but they will eventually lead to different VESTA surface cycles, and therefore, they are drawn twice here.

described further configurations. However, various extensions (*cf.*, Ref.s [12–14]) of the MCA use additional templates, which also use an additional average point for their triangular decomposition. In general, there is now a total of 14 different types of VESTA N -cycles ($N = 3, 4, 5, 6, 7, 8, 9, 12$). In particular, N -cycles with a length of 10 or 11 do not exist, since (complementary) N -cycles with a length of 2 or 1, respectively, do not exist either.

We would like to stress that VESTA can produce in total six different types of surfaces. There are the surfaces, which will be processed either in the global “disconnect” or “connect” modes. Then only VESTA N -cycles of length up to seven will be generated. A third

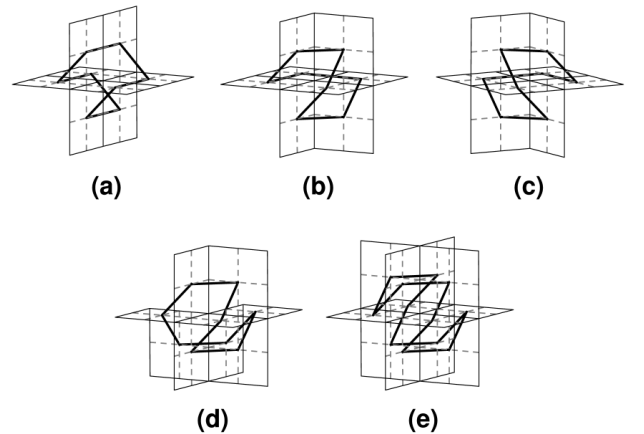


FIG. 12: Five more VESTA surface cycles (of maximum length) with corresponding boundary face chains: (a) – (c) nonplanar 8-cycles, (d) nonplanar 9-cycle, (e) nonplanar 12-cycle.

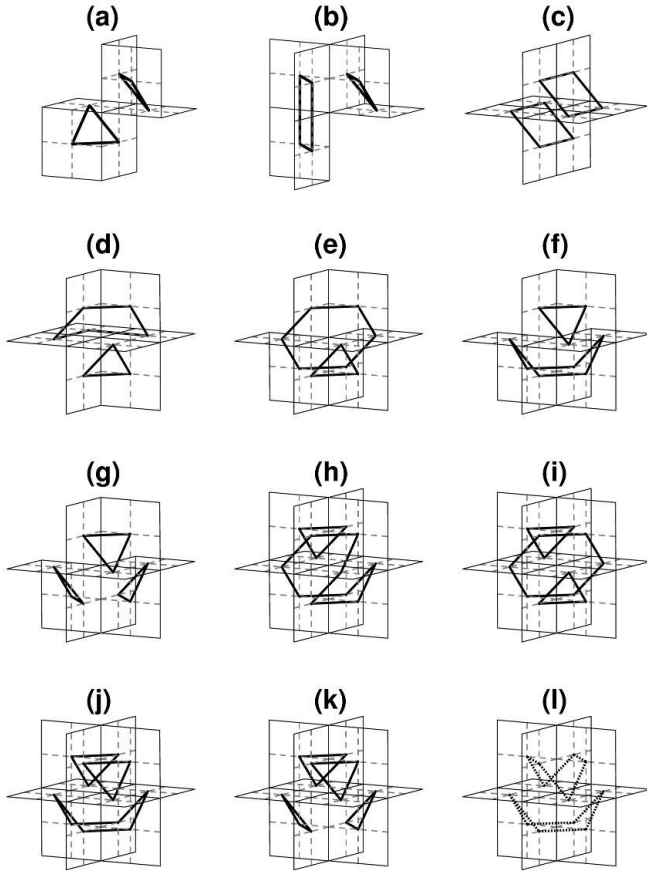


FIG. 13: Generation of multiple VESTA surface cycles: (a) results from Fig. 5.g; (b) results from Fig. 5.e; (c) results from Fig. 11.a; (d) results from Fig. 11.b or 11.c; (e) – (g) result from Fig. 11.d; (h) – (k) result from Fig. 11.e; (l) can never be created (Ref. [12] provides an exhaustive discussion).

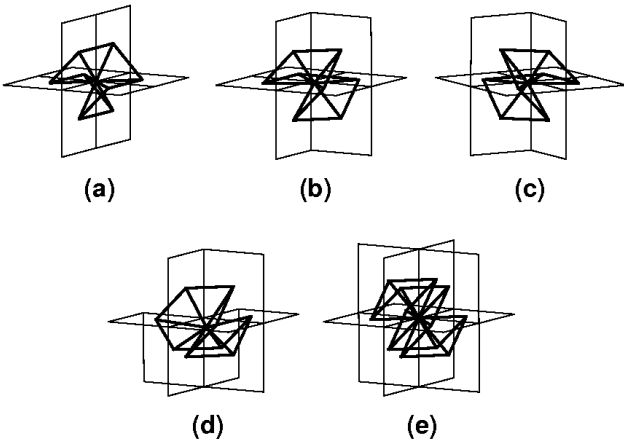


FIG. 14: The VESTA surface cycles as shown in Fig. 12 with decomposition into triangles: (a) – (c) 8-cycles, (d) 9-cycle, (e) 12-cycle. Here, average points are used to generate a surface tiling with triangles.

kind is generated while using the “mixed” mode, which has been described in this subsection. Furthermore, each of the three modes can be either executed in a so-called low resolution mode (“L”) where no additional points are used for the decomposition of the surface cycles; or in a so-called high resolution mode (“H”) where additional average points are used for the decomposition of surface cycles. Up to this point, we have described what we will later refer to as the “original VESTA”. Note, that this type of VESTA has been implemented successfully into efficient software [16]. In the following subsection, we shall discuss a marching variant of VESTA.

F. The Marching 3-Cell Version of VESTA

Using the original VESTA, one processes the 3D data set under consideration at once. This has the advantage that redundant information can be mostly avoided

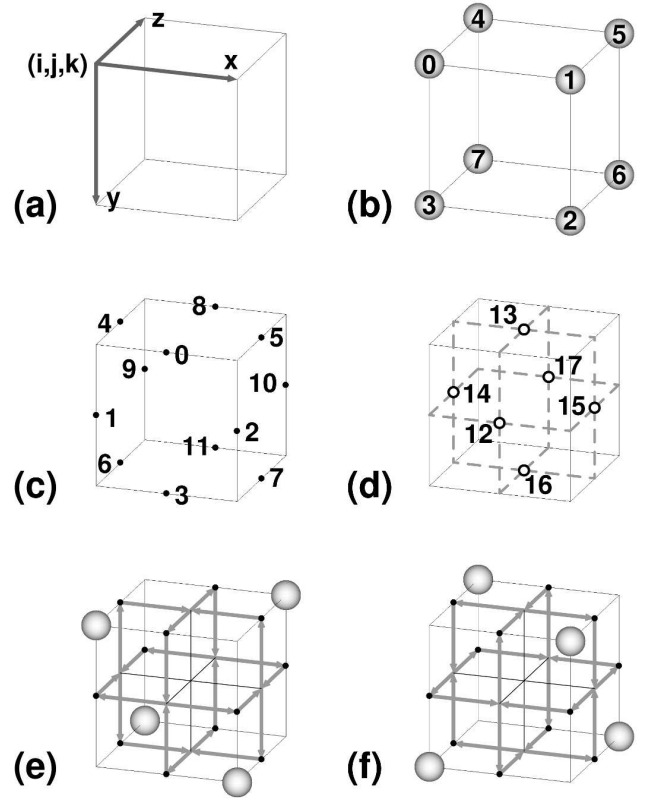


FIG. 15: Various conventions for the marching VESTA for a given 3-cell: (a) a marching 3-cell at position (i, j, k) , superimposed with a cartesian coordinate system, $x|y|z$; (b) numbering of voxels; the spheres mark active voxel centers, or “sites”; (c) numbering of VESTA surface support points, i.e., boundary face centers; (d) numbering of junctures, which may or may not become points of ambiguity; the dashed lines represent pairs of anti-parallel voxel face vectors; (e) voxel face vectors for voxels no.s 0, 2, 5, and 7; (f) voxel face vectors for voxels no.s 1, 3, 4, and 6; in (e) and (f) the black dots represent boundary face centers.

during the generation of surface tiles. However, a clear disadvantage is that a large amount of computer memory may be required, in particular, if large amounts of voxels have to be processed. As a consequence, a computer with insufficient RAM may be forced to go into a swapping mode, and the execution time may increase considerably. Therefore, we shall describe in the following an alternate implementation of VESTA, which we will name the “marching VESTA”.

In this subsection, we are going to describe the details of a scan of a given 3D data set with a marching $2 \times 2 \times 2$ voxel neighborhood (3-cell, or cube). All of the surface tiles, which finally will represent a single or multiple VESTA surface(s), will be generated and collected during such a scan. Note, that this scanning of the data is the same as in case of the MCA [4, 18]. However, we would like to stress, that we are confident to use such an approach only, because the ansatz of using the active voxels’ boundary faces as shown in Fig. 2.b has resulted in VESTA surface N -cycles, which are all confined to 3-cells (*cf.*, Figs 7 and 12). In the following, we shall outline the necessary steps for a more memory efficient VESTA implementation while using a few examples.

In Fig. 15.a, a marching 3-cell at position (i, j, k) is shown. The indices i , j , and k , refer to the position of voxel no. 0 (*cf.*, Fig. 15.b) in the x -, y -, and z -directions, respectively. Note that the coordinate system, $x|y|z$, coincides with the range vectors (*cf.*, Fig. 2) of voxel no. 0 for positive x -, y -, and z -directions. The VESTA surface support points (*cf.*, Fig. 15.c) can move within the bounds of the range vectors, which coincide with the edges of the marching 3-cell. Since an active voxel may show up in a marching 3-cell only with an eighth of its volume (i.e., an octant), its initial corresponding boundary faces may show up only as quadrants (*cf.*, Fig. 2).

In Table 1, the paths of VFV pairs are listed, which

Center ID	\oplus Path	\ominus Path
0	13 \rightarrow 0 \rightarrow 12	12 \rightarrow 0 \rightarrow 13
1	12 \rightarrow 1 \rightarrow 14	14 \rightarrow 1 \rightarrow 12
2	15 \rightarrow 2 \rightarrow 12	12 \rightarrow 2 \rightarrow 15
3	12 \rightarrow 3 \rightarrow 16	16 \rightarrow 3 \rightarrow 12
4	14 \rightarrow 4 \rightarrow 13	13 \rightarrow 4 \rightarrow 14
5	13 \rightarrow 5 \rightarrow 15	15 \rightarrow 5 \rightarrow 13
6	16 \rightarrow 6 \rightarrow 14	14 \rightarrow 6 \rightarrow 16
7	15 \rightarrow 7 \rightarrow 16	16 \rightarrow 7 \rightarrow 15
8	17 \rightarrow 8 \rightarrow 13	13 \rightarrow 8 \rightarrow 17
9	14 \rightarrow 9 \rightarrow 17	17 \rightarrow 9 \rightarrow 14
10	17 \rightarrow 10 \rightarrow 15	15 \rightarrow 10 \rightarrow 17
11	16 \rightarrow 11 \rightarrow 17	17 \rightarrow 11 \rightarrow 16

TABLE I: Directed paths for the quadrants of the oriented boundary faces, which have their centers at the predefined locations as depicted in Fig. 15.c. The start and end points of the 24 paths are the junctures, which are shown in Fig. 15.d.

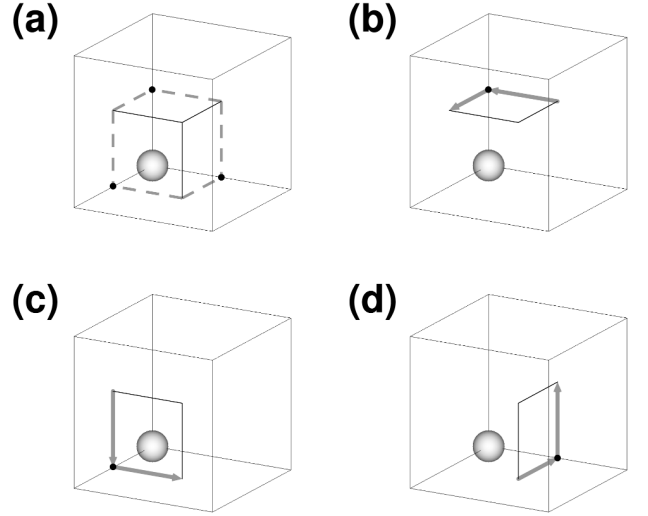


FIG. 16: A 3-cell with one single active voxel (sphere, representing its voxel center) together with the (a) corresponding octant of the active voxel volume, (b) - (d) the three different, contributing boundary face quadrants, i.e., the boundary face centers (black dots) and two voxel face vectors (gray) each.

each correspond to a quarter (i.e., a quadrant) of a potential boundary face of an active voxel within the 3-cell (i, j, k) . These paths connect junctures (*cf.*, Fig. 15.d) via the boundary face centers (*cf.*, Fig. 15.c) for the given boundary faces. Since each boundary face is shared by two side by side voxels, two orientations, \oplus and \ominus , exist for the VFV based paths with respect to the positive and negative spatial directions. In Figs 15.e and 15.f together, all 48 possible VFVs are shown for the marching 3-cell.

As an example, in Fig. 16.a, a 3-cell is shown with the single active site, no. 7, together with an octant of the active voxel, representing its partial volume. The three black dots in the figure represent the corresponding boundary face centers, and the six dashed lines connect these with the corresponding junctures. I.e., for a single active site, only three boundary face quadrants have to be considered at maximum. Note, that each of the boundary face quadrants are oriented. In Figs 16.b, 16.c, and 16.d, the corresponding (gray) VFV pairs are drawn, together with their corresponding boundary face centers, i.e., the paths $9\ominus$, $6\ominus$, and $11\oplus$ (*cf.*, Table 1), respectively. The three paths connect to the closed point sequence, $17 \rightarrow 9 \rightarrow 14 \rightarrow 6 \rightarrow 16 \rightarrow 11 \rightarrow 17$. After the removal of points with identities > 11 (i.e., junctures), the sequence will transform into the VESTA 3-cycle, $9 \rightarrow 6 \rightarrow 11 \rightarrow 9$. As a result, we obtain a single oriented triangle within the 3-cell.

In another example, we shall process the three active sites as shown in Fig. 17. First, all of the contributing boundary face quadrants are determined with their proper orientation. In Fig. 17.a, the according seven boundary face quadrants are shown, together with the

corresponding boundary face centers. In a next step, one considers all relevant VFV paths (*cf.*, Fig. 17.b). If we focus on the six faces of the 3-cell, we notice that a face of a 3-cell can always contain either zero, two, or four VFVs. In this given example, five of the six 3-cell faces contain only two VFVs each. With the help of Table 1, the connection of the vectors is straightforward here.

But, one 3-cell face contains four VFVs, and at its center we observe a POA. Note that for the POA the application of the connection diagram as shown in Fig. 1.d will suffice for the proper execution of a previously specified “disconnect” and/or “connect” mode. In other words, for the marching VESTA, the much more complicated usage of the four connection diagrams as shown in Fig. 6.b is not necessary, hence, lesser computational decisions have to be made. Finally, considering the chosen connectivities among the VFVs, and after omission of the junctures, we end up with two solutions for the VESTA surface cycles. We obtain either a single 3-cycle and a single 4-cycle that are resulting from the “disconnect” mode (*cf.*, Fig. 17.c), or we obtain a single 7-cycle that results from the “connect” mode (*cf.*, Fig. 17.d).

For the sake of completeness, we demonstrate in Fig. 18 for two active sites the recommended processing steps for boundary face center displacement (which is in general necessary for isosurface generation, *cf.*, Fig.s 18.a and 18.b), and the breakup of the VESTA N -cycles ($N > 3$) into 3-cycles (*cf.*, Fig.s 18.c – 18.f). In this given example, we have one single VESTA 4-cycle. Note, that the contributing range vectors coincide with the respective edges of the 3-cell (*cf.*, Fig.s 18.a and 18.b). Further-

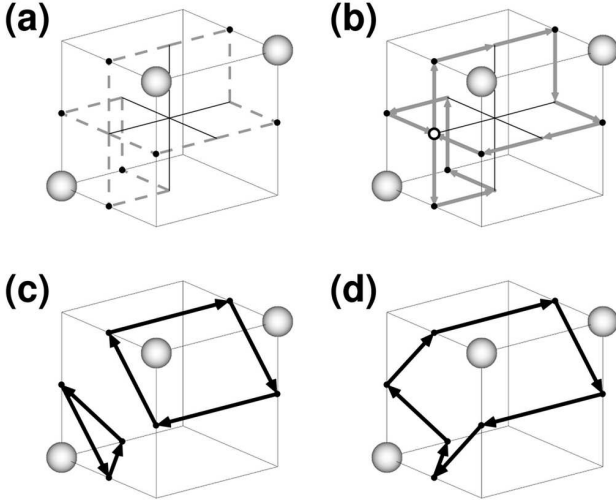


FIG. 17: A 3-cell with three active voxels (spheres, representing their voxel centers) together with (a) the corresponding octants of the active voxel volumes, (b) the various contributing boundary face quadrants, seven boundary face centers (black dots), fourteen voxel face vectors (gray), and a single point of ambiguity (white dot), (c) a single 3-cycle and a single 4-cycle that are resulting from the “disconnect” mode, and (d) a single 7-cycle that results from the “connect” mode.

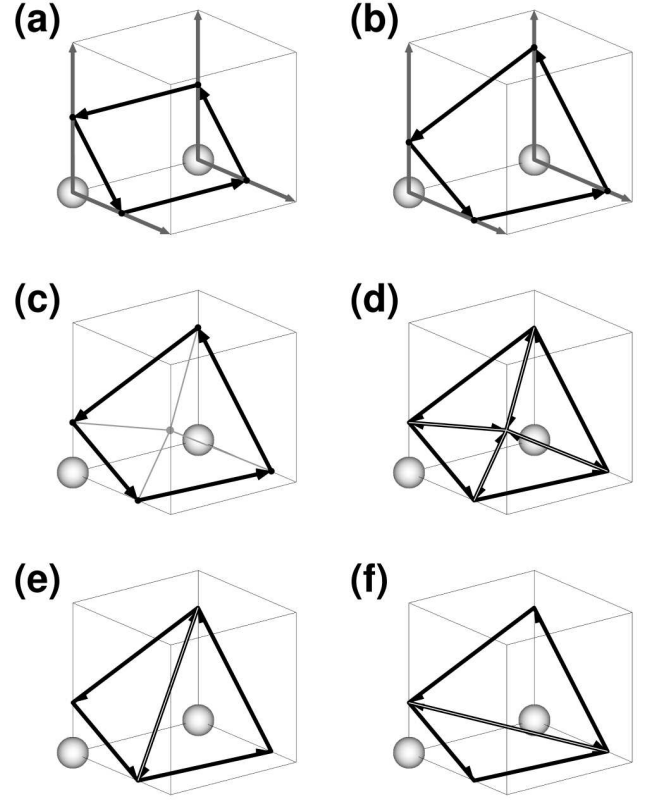


FIG. 18: (a) A 3-cell with two active voxels (spheres, representing their voxel centers) together with the corresponding range vectors and a resulting 4-cycle; (b) as in (a), but with displaced boundary face centers; (c) as in (b), but without the range vectors and an additional average point (gray) and further line segments (gray) indicating the anticipated triangle decomposition; (d) as in (c) with four 3-cycles instead of a single 4-cycle; (e) a convex-shaped, and (f) a concave-shaped solution without the usage of additional support points. Note, that the orientations of the newly formed 3-cycles are inherited from the initially given 4-cycle.

more, it is recommended that the boundary face centers will be displaced before the breakup into 3-cycles, because then one is not required to reevaluate the 3D position of the possibly inserted average point, which once again may save computing time. Fig.s 18.e and 18.f illustrate the introduction of biases for the shape of the generated surface section, if no additional support points, e.g., average points, will be used.

This concludes the description of the marching VESTA.

G. VESTA vs. the Marching Cubes Algorithm

In this subsection, we provide an in-depth comparison between VESTA and the original MCA[4], because the original MCA can produce holes in the final surfaces, whereas VESTA does not. Since the marching VESTA scans the given 3D data set exactly like the MCA, we

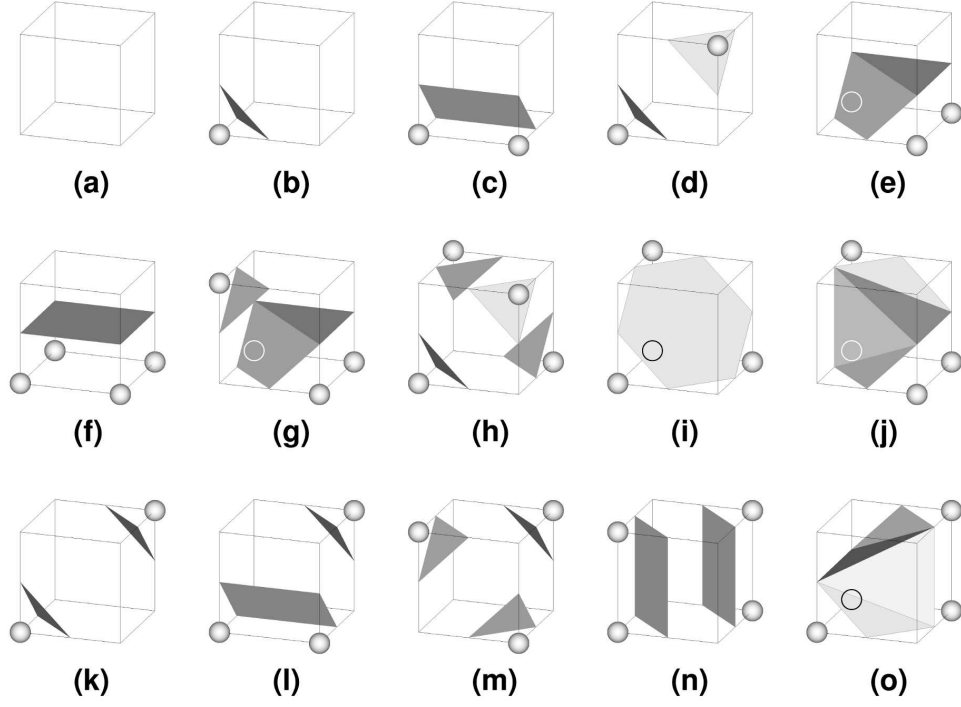


FIG. 19: Full surface template set of the original Marching Cubes algorithm. White and black circles indicate blanketed active 3-cell sites.

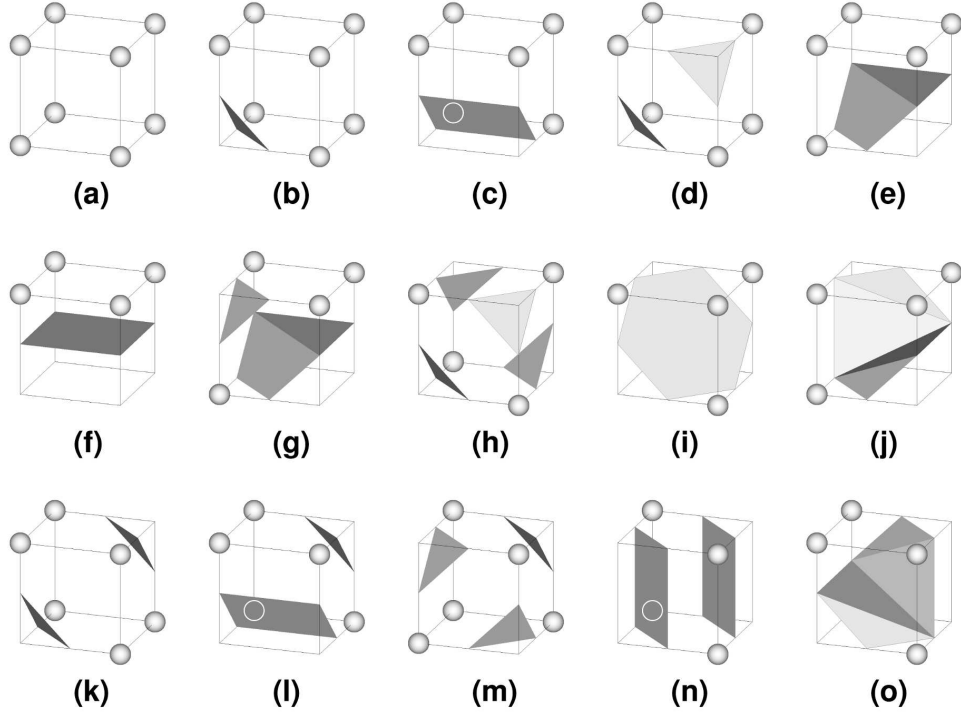


FIG. 20: As in Fig. 19, but with inverted 3-cell sites.

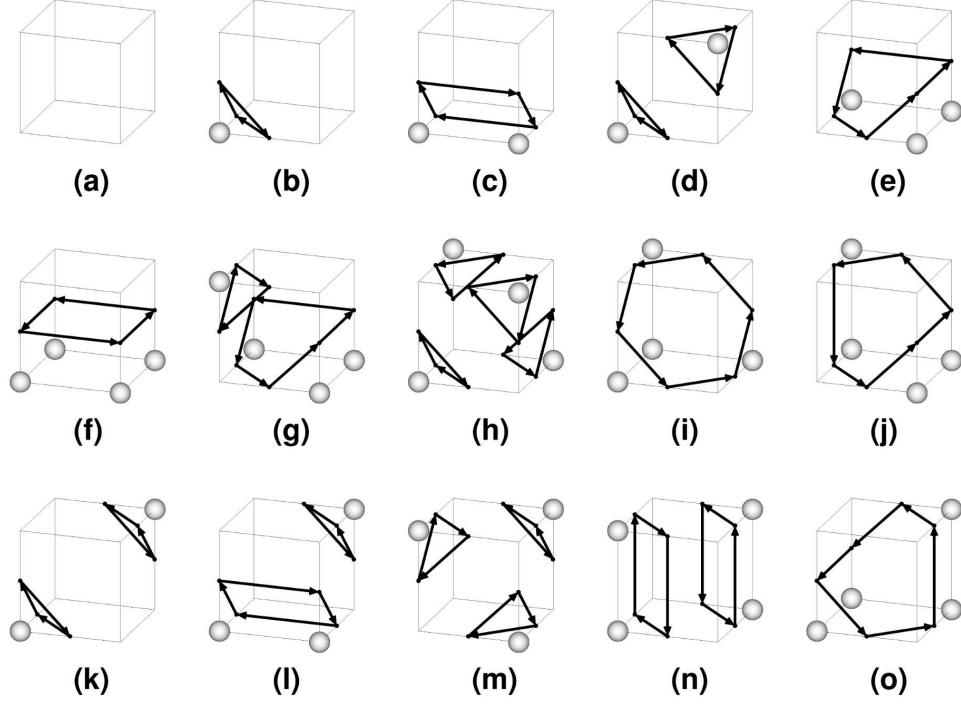


FIG. 21: VESTA surface cycles in “disconnect” mode, for the 3-cells shown in Fig. 19.

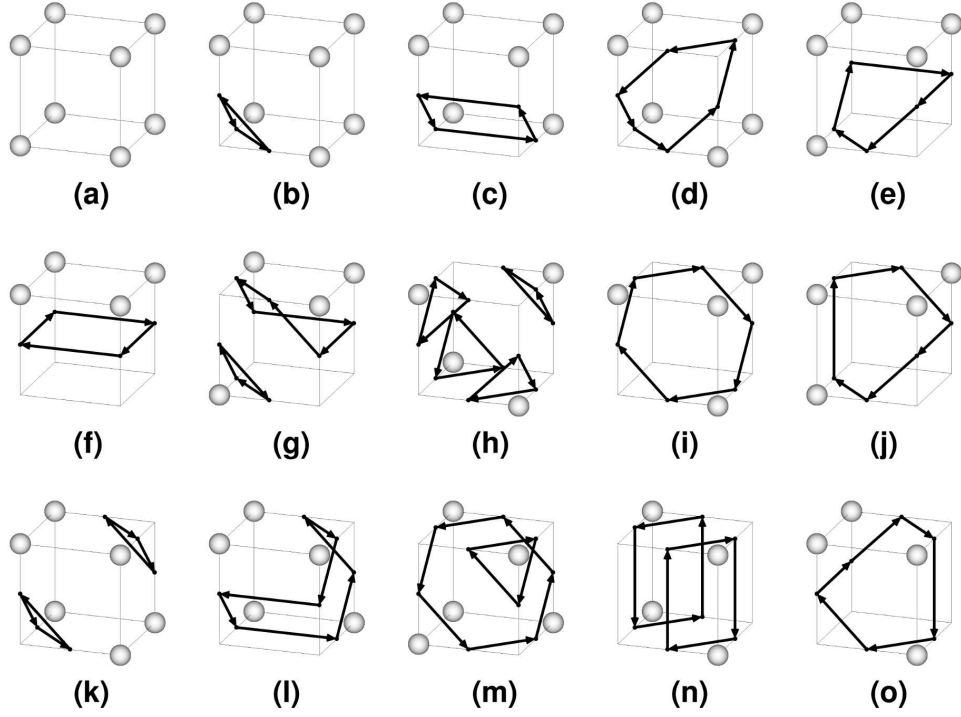


FIG. 22: VESTA surface cycles in “disconnect” mode, for the 3-cells shown in Fig. 20.

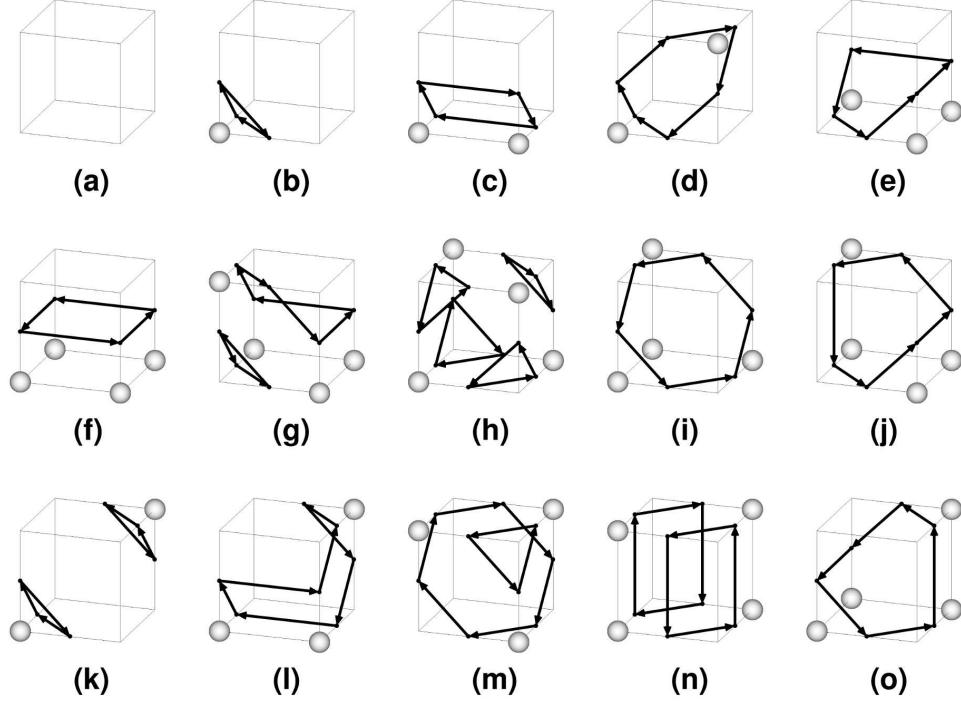


FIG. 23: VESTA surface cycles in “connect” mode, for the 3-cells shown in Fig. 19.

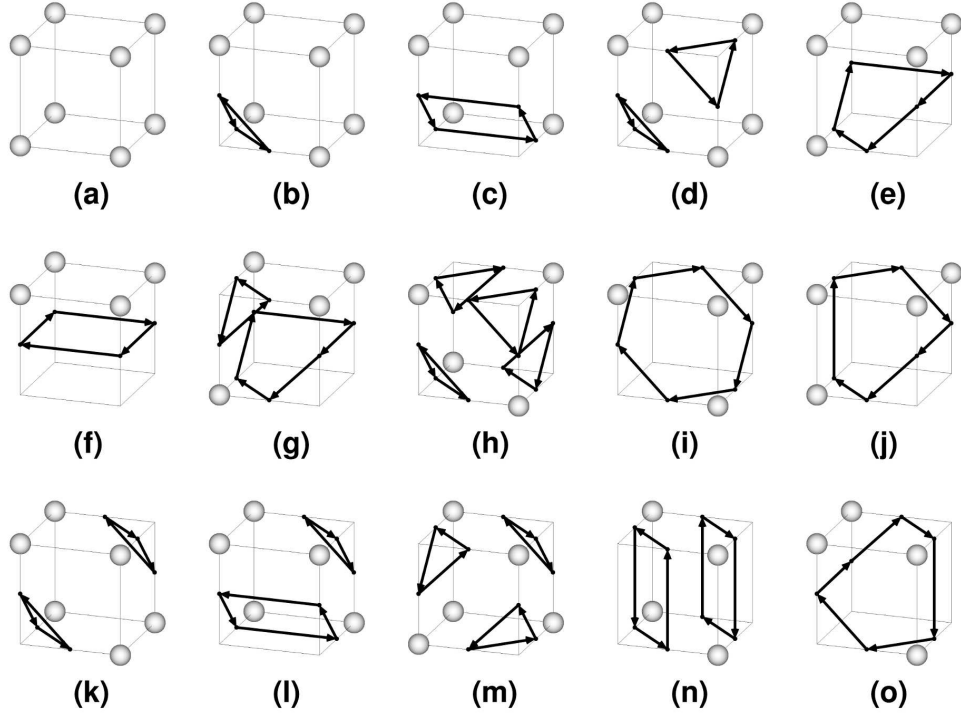


FIG. 24: VESTA surface cycles in “connect” mode, for the 3-cells as shown in Fig. 20.

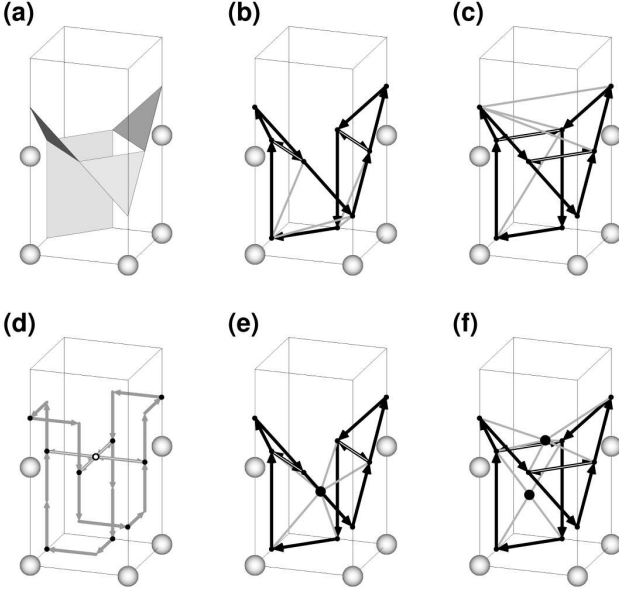


FIG. 25: Comparison between the Marching Cubes algorithm and VESTA for two side-by-side 3-cells: (a) a hole is created due to the original MCA's limited template set; (b) “disconnect” mode solution for VESTA with low (“L”) resolution cycle decomposition; (c) “connect” mode solution for VESTA with L-decomposition; (d) voxel face vectors (gray), boundary face centers (black dots) and a single point of ambiguity (white dot); (e) as in (b), but with high (“H”) resolution cycle decomposition; (f) as in (c), but with H-decomposition.

can compare the results of VESTA 3-cell by 3-cell with the various templates, which the original MCA uses. Since VESTA can distinguish between a “disconnect” and a “connect” mode whenever POAs are encountered, we shall present both solutions for all given 3-cells.

In Fig. 19, the complete surface section template set of the original MCA is shown for its 15 configurations of different 3-cell site occupancies (*cf.*, also, e.g., Ref.s [2, 5]). The original MCA uses the same set of templates for its correspondingly inverted 3-cell sites (*cf.*, Fig. 20). In comparison to Fig. 19, we show the computed surface N -cycles for VESTA in global “disconnect” (“connect”) mode in Fig. 21 (Fig. 23); and in comparison to Fig. 20, we show the computed surface N -cycles for VESTA in global “disconnect” (“connect”) mode in Fig. 22 (Fig. 24). Several differences can be observed.

First, we provide a comparison between VESTA in its global “disconnect” mode and the original MCA. VESTA reproduces in Fig. 21 perfectly the perimeters of the MCA template set (*cf.*, Fig. 19). However, if we compare Fig. 22 with Fig. 20, there are differences between the cases (d), (g), (h), (l), (m), and (n). In particular, the original MCA does not provide templates like the processed VESTA N -cycles shown in Figs. 8.e and 8.g.

Secondly, we provide a comparison between VESTA in its global “connect” mode and the original MCA.

VESTA reproduces in Fig. 24 perfectly the perimeters of the MCA template set for inverted sites (*cf.*, Fig. 20). However, if we compare Fig. 23 with Fig. 19, there are differences – once again – between the cases (d), (g), (h), (l), (m), and (n). As already pointed out, the original MCA misses templates like the processed VESTA N -cycles shown in Figs. 8.e and 8.g.

Finally, we show in Fig. 25.a an example where the combination of the original MCA's templates leads to the creation of a hole in the surface. On the contrary, VESTA consistently applies the “disconnect” and/or “connect” modes for the two side-by-side 3-cells under consideration, and it does not create any holes (*cf.*, Figs. 25.b and 25.c). The reason for VESTA's success lies in the ansatz based on the VFVs of Fig. 2.b, and in the careful resolution of ambiguities at the junctures (*cf.*, Fig. 25.d). In Figs. 25.e and 25.f, VESTA's high resolution surface tilings are shown in addition for this particular example.

Apparently, it is wrong to just take the same template set also for inverted 3-cells. Nowadays, various extensions of the original MCA account for the missing surface templates (*cf.*, e.g., Ref.s [12, 14, 18]). In fact, some authors seem to provide too many additional surface templates (*cf.*, Ref. [13]), i.e., some of these are actually not necessary for an adequate surface extraction.

Instead of using many surface templates, VESTA only uses a single building block (*cf.*, Fig. 2.b) as the basis for surface construction. This concludes the theoretical section of this paper.

III. APPLICATIONS

In this application section, VESTA is used to create iso-surfaces for 3D image data that have been generated from CT-scans [21] and x-ray microtomography [22], respectively. Furthermore, it is used to create a freeze-out hypersurface from a set of 3D numerical simulation data [23, 24] that result from the field of theoretical relativistic heavy-ion physics. We also provide benchmarks between the original VESTA, the marching VESTA, and an extended computer code implementation [18] of the original MCA (*cf.*, “extended MCA”, in the benchmark tables).

The software by Bourke et al. [18] fixes the problems of the original MCA, and it uses a surface template set that exactly agrees to VESTA, when it is executed in a low resolution (“L”), global “disconnect” mode. In this particular mode, VESTA will generate surface N -cycles up to length seven only (*cf.*, Figs. 7 and 8). In order to be able to better compare the execution times of the codes under consideration, all software implementations have been prepared in the following ways.

All codes first load a full 3D data set into the computers' memory, and an initial isovalue has been provided. Then the start time is taken and the stop watch begins

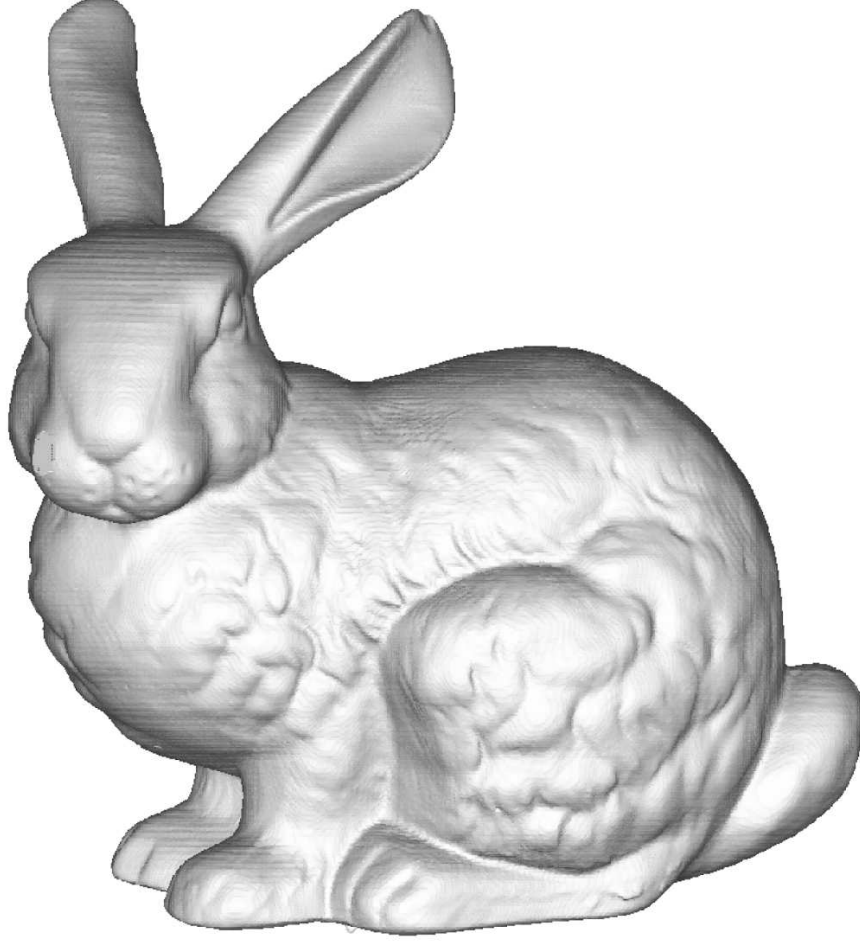


FIG. 26: VESTA isosurface rendering for CT-scan data of the Stanford Terra-Cotta Bunny [25].

Technique	original VESTA			marching VESTA			extended MCA
Connectivity	Disconnect	Connect	Mixed	Disconnect	Connect	Mixed	Disconnect
3-Cycles	254, 662	254, 662	254, 662	254, 662	254, 662	254, 662	254, 662
4-Cycles	550, 229	550, 229	550, 229	550, 229	550, 229	550, 229	550, 229
5-Cycles	178, 512	178, 512	178, 512	178, 512	178, 512	178, 512	178, 512
6-Cycles	38, 063	38, 063	38, 063	38, 063	38, 063	38, 063	38, 063
7-Cycles	0	0	0	0	0	0	0
8-Cycles	<i>N/A</i>	<i>N/A</i>	0	<i>N/A</i>	<i>N/A</i>	0	<i>N/A</i>
9-Cycles	<i>N/A</i>	<i>N/A</i>	0	<i>N/A</i>	<i>N/A</i>	0	<i>N/A</i>
12-Cycles	<i>N/A</i>	<i>N/A</i>	0	<i>N/A</i>	<i>N/A</i>	0	<i>N/A</i>
Cycle Sum	1, 021, 466	1, 021, 466	1, 021, 466	1, 021, 466	1, 021, 466	1, 021, 466	1, 021, 466
L: Points	1, 021, 460	1, 021, 460	1, 021, 460	4, 085, 840	4, 085, 840	4, 085, 840	6, 128, 724
L: Triangles	2, 042, 908	2, 042, 908	2, 042, 908	2, 042, 908	2, 042, 908	2, 042, 908	2, 042, 908
L: Time [s]	20.44(7)	20.48(5)	20.38(2)	29.62(34)	29.62(31)	29.51(30)	28.20(41)
H: Points	1, 788, 264	1, 788, 264	1, 788, 264	4, 852, 644	4, 852, 644	4, 852, 644	<i>N/A</i>
H: Triangles	3, 576, 516	3, 576, 516	3, 576, 516	3, 576, 516	3, 576, 516	3, 576, 516	<i>N/A</i>
H: Time [s]	20.44(2)	20.88(2)	20.86(7)	30.86(44)	30.30(45)	30.44(46)	<i>N/A</i>

TABLE II: Stanford Terra-Cotta Bunny CT-scan data benchmark: 361 images with dimensions $512 \times 512 = 262,144$ pixels each; isovalue equals to 150; number of active voxels: 3,100,197, i.e., 3.276%.

to run. The codes perform their various tasks and create lists of surface support points and surface triangles, respectively. Before all of the generated data are stored into a file that eventually may be used for, e.g., rendering purposes, the stop watch is halted and the stop time is taken. The time, which is the difference time between the stop and start times, is listed as an average in Tables 2 – 4 for a total of 100 runs each.

Note that for this benchmark an Amilo notebook by Fujitsu Siemens Computers has been used. Its hardware consists of a Pentium®Dual-Core CPU T4200 @ 2.00GHz, and 4.00GB RAM. The disk operating system of this computer is an Ubuntu 10.04 LTS Linux distribution.

A. Processing of CT-Scan Data of the Stanford Terra-Cotta Bunny

As a first application, we present in Fig. 26 a VESTA isosurface rendering for CT-scan data of the Stanford Terra-Cotta Bunny [25]. Note, that this particular rendering uses a flat shading (*cf.*, Ref. [3]) for the 150-valued isosurface, which has been generated while using VESTA in a high resolution (“H”) “mixed” mode (the latter uses gray-level information to determine locally, whether a “disconnect” or a “connect” mode should apply).

In Table 2, all benchmark information is listed. In particular, all three considered versions of codes yield the same results with respect to the numbers of surface cycles. The bunnies’ surface is apparently smooth enough, so that N -cycles up to length six suffice for an adequate result. Note that the MCA produces the largest amount of redundant information, i.e., three support points for each created surface triangle, whereas the original VESTA produces always the minimum of the required support points. The marching VESTA avoids in our particular implementation redundant information only within each given 3D data scanning 3-cell.

Postprocessing for the removal of redundant support point information has not been performed by us, since we intended to make only minimal adaptations to the source code of the MCA implementation by Bourke et al. [18]. Therefore, this according effort has not been put into the marching VESTA implementation either, in order to be able to provide benchmark results that allow for a better comparison among the codes. As a result, the here used MCA and marching VESTA codes execute somewhat faster than they normally would, if redundant support points were removed. Note that the original VESTA executes the fastest for this particular data set (*cf.*, the bold faced “L”-times in Table 2).

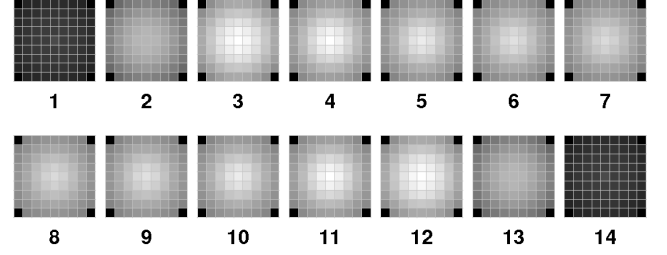


FIG. 27: 3D image data set, comprised of an ordered and numbered stack of 14 2D images with 9×9 pixels each.

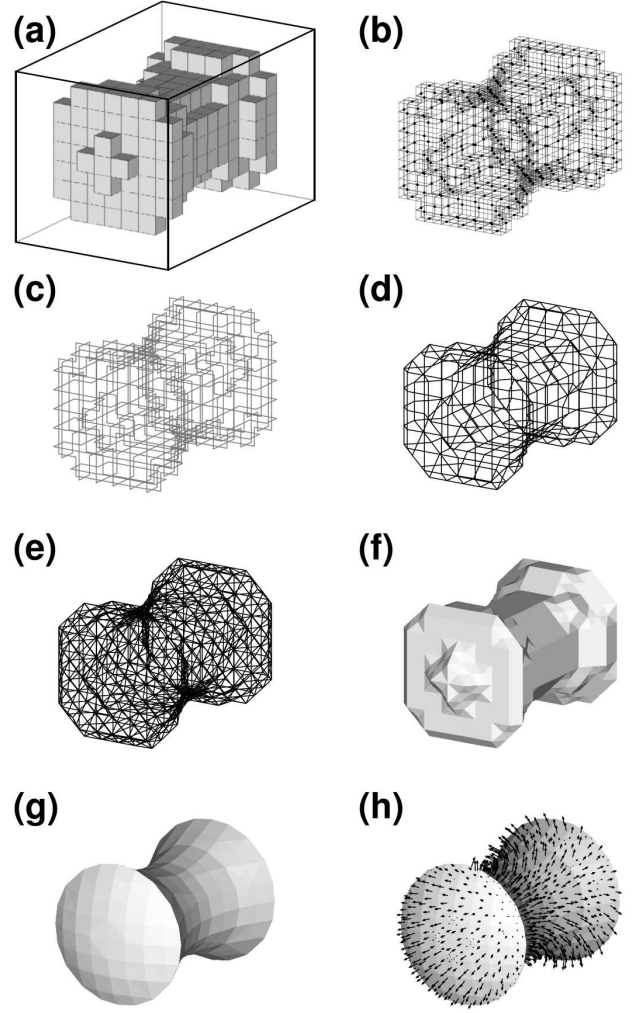


FIG. 28: (a) Selected voxels within the $9 \times 9 \times 14$ voxel data set as shown in Fig. 27; (b) initial boundary faces with their centers (black dots) and voxel face vector pairs (gray lines); (c) voxel face vector wire frame; (d) VESTA wire frame for the selected voxels shown in (a); (e) wire frame after the breakup of the N -cycles into 3-cycles; (f) surface rendering for the wire frame shown in (e); (g) isosurface rendering using voxel gray-levels for the support point displacement; (h) as in (g) with superimposed surface triangle normal vectors.

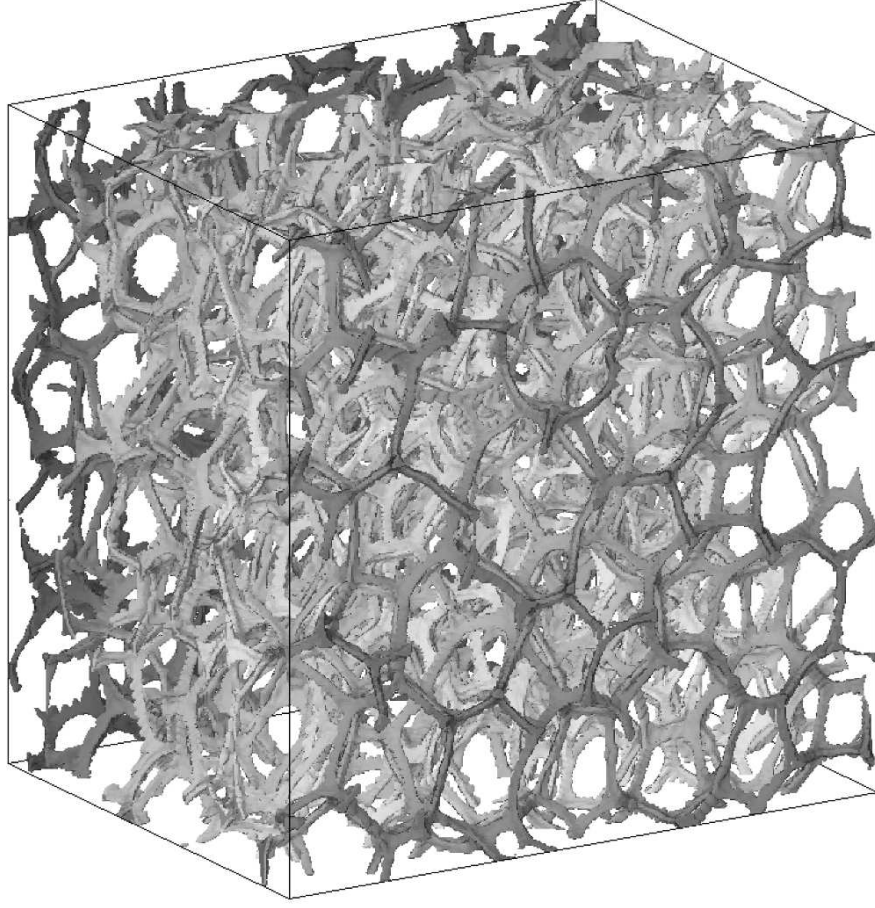


FIG. 29: VESTA isosurface rendering of a foam from 3D image data [26] generated with x-ray microtomography.

Technique	original VESTA			marching VESTA			extended MCA
Connectivity	Disconnect	Connect	Mixed	Disconnect	Connect	Mixed	Disconnect
3-Cycles	273, 988	272, 322	275, 741	273, 988	272, 322	275, 741	273, 988
4-Cycles	415, 837	419, 801	417, 010	415, 837	419, 801	417, 010	415, 837
5-Cycles	185, 836	185, 836	185, 066	185, 836	185, 836	185, 066	185, 836
6-Cycles	37, 749	40, 564	37, 804	37, 749	40, 564	37, 804	37, 749
7-Cycles	8, 300	4, 336	5, 589	8, 300	4, 336	5, 589	8, 300
8-Cycles	N/A	N/A	1, 539	N/A	N/A	1, 539	N/A
9-Cycles	N/A	N/A	26	N/A	N/A	26	N/A
12-Cycles	N/A	N/A	0	N/A	N/A	0	N/A
Cycle Sum	921, 710	922, 859	922, 775	921, 710	922, 859	922, 775	921, 710
L: Points	932, 739	932, 739	932, 739	3, 699, 086	3, 699, 086	3, 700, 651	5, 566, 998
L: Triangles	1, 855, 666	1, 853, 368	1, 856, 666	1, 855, 666	1, 853, 368	1, 856, 666	1, 855, 666
L: Time [s]	5.34(1)	5.20(1)	5.30(1)	4.62(26)	4.66(26)	4.58(26)	4.71(34)
H: Points	1, 580, 461	1, 583, 276	1, 579, 773	4, 346, 808	4, 349, 623	4, 346, 120	N/A
H: Triangles	3, 151, 110	3, 154, 442	3, 147, 604	3, 151, 110	3, 154, 442	3, 147, 604	N/A
H: Time [s]	5.23(1)	5.17(1)	5.12(1)	5.18(36)	5.17(36)	5.19(36)	N/A

TABLE III: X-ray micro tomography data benchmark: 200 images with dimensions $256 \times 256 = 65,536$ pixels each; isovalue equals to 135; number of active voxels: 512,603, i.e., 3.911%.

B. Isosurface Rendering for X-Ray Microtomographic Data

In the field of x-ray microtomography [22], like tomography [21], one creates cross-sections during scans of a 3D object. These cross-sections are 2D images with pixel sizes in the micrometer range. The stacking of the 2D images results in a 3D image (data set). Such a 3D image represents a virtual model of the original object. Hence, x-ray microtomography provides a way to create a virtual model from an object without destroying it.

Let us begin with the sequence of 14 2D gray-level images with 9×9 pixels, which is shown in Fig. 27. After their stacking, they represent a 3D image. The pixels here assume gray values in the range from 0 to 255. These bounds represent black and white, respectively. In the following, we shall apply VESTA to this 3D image in order to render an isosurface with isovalue 180. First, all voxels that have a gray value larger or equal to 180 are selected. The cluster of selected voxels is shown in Fig. 28.a, together with the total volume of the $9 \times 9 \times 14$ voxels. Note that this initial 3D shape of selected voxels is already enclosed with a surface that has no holes at all; namely the union of all boundary faces that enclose all active voxels.

Here we shall use gray-level information to automate the (consistent) local choosing of the “disconnect” and the “connect” mode, respectively. If two voxels that ought to be enclosed have only one single voxel edge in common, we determine the (here linearly interpolated) gray value of corresponding point of ambiguity. This gray value is evaluated as the average of the gray-levels of the four voxels, which share this common edge.

After application of VESTA, but without support point displacement, one obtains – as intermediate steps – the results as illustrated in Figs 28.b – 28.d. The wire frame in Fig. 28.d represents the union of all VESTA N -cycles where $N = 3, \dots, 12$. In Fig. 28.e, a much denser wire frame is shown, which is the result of the break down of the N -cycles ($N > 3$) into 3-cycles while making use of additional average points as described in the theoretical section (*cf.*, Figs 9 and 14). In a next step, the VESTA 3-cycles are depicted as solid triangles (*cf.*, Fig. 28.f). Note that the shade of gray of each triangle is determined through the evaluation of its normal vector.

The preliminary VESTA surface as shown in Fig. 28.f looks somewhat bulky. However, we should stress that all VESTA surfaces are at this stage of the processing perfect in the sense, that they are non-degenerate, i.e., they always enclose a volume that is larger than zero, and they never self-intersect and/or overlap each other. Furthermore, the information of the interior/exterior of the enclosed shapes is propagated correctly at all times.

In a final step, we generate an isosurface from the VESTA surface. This is done by displacement of all voxel centers within the bounds defined by their range vectors, r , (*cf.*, Fig. 2) and by subsequent reevaluation of the average points, which have been used for the N -

cycle breakup. In particular, the gray-levels of the two voxels, which define a given range vector, r , are interpolated linearly here (note that other techniques may be applied [19, 20]).

The final isosurface representing an isovalue of 180 for the given 3D image data is shown in Fig. 28.g. And in Fig. 28.h, this isosurface is superimposed with normal vectors of the visible triangles. Note that this isosurface is quite smooth considering the rather coarse granularity of the underlying 3D voxel space. Furthermore, the directions of the normal vectors of the surface elements, i.e., triangles, are not limited to the six directions of the normals of the initial boundary faces.

While using the processing steps as outlined in this subsection, in Fig. 29, the rendering of a VESTA isosurface with isovalue 135 is shown for 3D x-ray microtomographic image data of a metallic foam [26]. In order to better perceive the depth of the data, the rendered surface has been color encoded prior to its transformation into shades of gray. The corresponding benchmark results are presented in Table 3. Note that the original VESTA needs a little more computing time than the marching VESTA and the MCA codes, however it generates significantly lesser surface support points.

C. Freezeout Hypersurface Extraction for Expanding Fireballs

One of the foremost objectives within the field of heavy-ion physics is the exploration of the equation of state (EOS) of nuclear matter (*cf.*, Ref. [27] and references therein). In heavy-ion physics experiments so-called “fireballs” are created, which are very hot and dense zones of nuclear matter. The theoretical description of these fireballs includes – but is not limited to – a relativistic hydrodynamic component, in which an EOS has to be explicitly postulated (*cf.*, e.g., Ref.s [28, 29]). The resulting space-time evolution of the fireball is driven by the EOS, and hence, is expected to manifest itself in calculated multi-particle production spectra that will eventually be compared to experimental data.

In some theoretical descriptions (*cf.*, e.g., Ref.s [30, 31]), the space-time evolution of the (ultra-relativistic) fluids is in part represented through a so-called “freezeout hypersurface” (FOHS), which is an isosurface within space-time, e.g., with respect to the temperature of the fluid. The FOHS has to be extracted from the hydrodynamic simulation data so that further (numerical) calculations can be performed. Let us consider here the $2 + 1$ dimensional relativistic hydrodynamic simulation code HYLANDER-C [32, 33] that can be used to study radially symmetric, so-called “central”, heavy-ion collisions.

The numerical HYLANDER-C simulations are performed on a cartesian grid in the two spatial (2D) variables, r and z , and in the temporal (+ 1D) variable

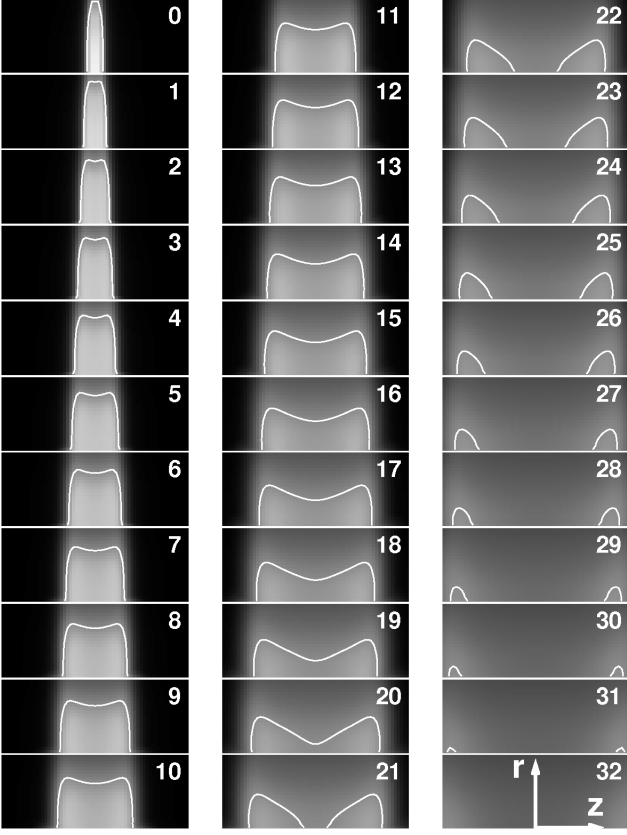


FIG. 30: A temporal sequence of temperature fields as a function of the longitudinal direction, z , and the radius, r . Each frame (except the latter) is superimposed with one or two (white) freezeout isocontours with isovalue $T_f = 139\text{MeV}$ (see text).

t . In particular, r denotes the radius of the system, z the longitudinal direction, and t the time, respectively. A solution of the hydrodynamic equations may be viewed as 3D image data where each voxel contains continuous values for each of the various physical field quantities under consideration, such as temperature, T , energy density, ϵ , components of the fluid velocity \vec{u} , etc. Before we treat 3D simulation data, we shall first take a look at a temporal sequence of spatial 2D data.

In Fig. 30, a temporal sequence of 33 temperature fields is shown as a function of the longitudinal direction, z , and the radius, r . The time steps remain the same between two successive frames, which are numbered from 0 to 32. The hydrodynamic grid has originally a much higher resolution than the here shown down-sampled 2D images. Each 2D frame in the figure has a resolution of $61 \times 24 = 1,464$ pixels, but eventually we shall process a slightly larger 3D data set (*cf.*, Table 4). Note that this lower resolution provides sufficient numerical accuracy for the desired FOHS in this given example. Black pixels correspond to a fluid temperature of 0MeV , whereas white pixels refer to a fluid temperature of 255MeV and/or higher.

For all frames DICONEX isocontours with isovalue (“freezeout temperature”, $T_f =$) 139MeV have been extracted (white contours). The last frame, no. 32, does not contain any isocontour, since all of its pixels have temperatures below T_f . Note that unphysical line segments have been removed where (at least) one end point has a radial value that is smaller than zero (*cf.*, also Ref. [17] on the extraction of a FOHS in 2D).

If one stacks up the DICONEX freezeout contours of Fig. 30 in 3D with the intent to construct a surface (*cf.*, Fig. 31.a), one requires both orientations of the contours. The latter is a consequence of the fact that in 3D one can look upon a 2D pixel in two ways, e.g., from “above” and from “below”. However, if we consider in Fig. 30 the transition from frame no. 20 to frame no. 21, we observe a *correspondence problem*. Apparently, a single isocontour has to be transformed into two isocontours, but it is unclear, how it can be accomplished within this approach.

Figs 31.b and 31.c offer the two alternate ways to build stacks of isocontours from the other possible projections within the 3D space under consideration. Again, unphysical line segments have been removed in the figures where (at least) one end point has a radial and/or a temporal value that is smaller than zero. For our particular example, no correspondence problems occur this time, but it cannot be ensured for different scenarios. Note that before the invention of VESTA in the year 2002 [15], FOHS construction within HYLANDER-C was initiated while using stacks

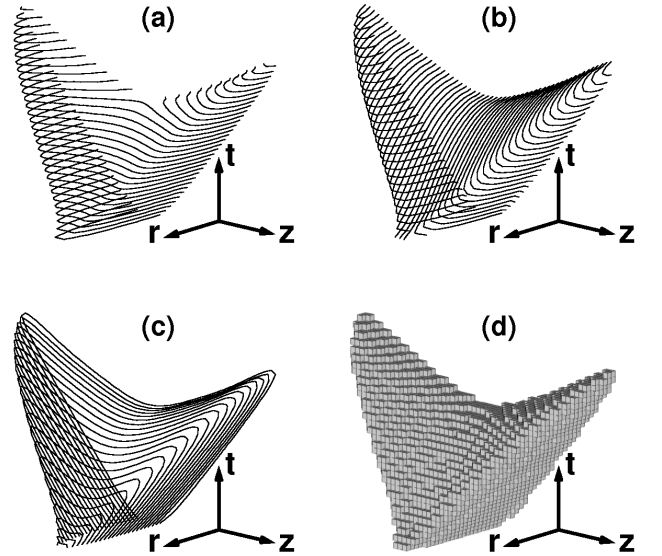


FIG. 31: (a) - (c) Stacked DICONEX freezeout contours, (a) of the $z-r$ planes at fixed t values, (b) of the $r-t$ planes at fixed z values, and (c) of the $z-t$ planes at fixed r values; note, that each line segment of the contours consists of two antiparallel vectors, since in 3D both orientations are necessary when building a surface; (d) the 3D stack of voxels, which each represent a temperature that is equal to or higher than T_f (see text).

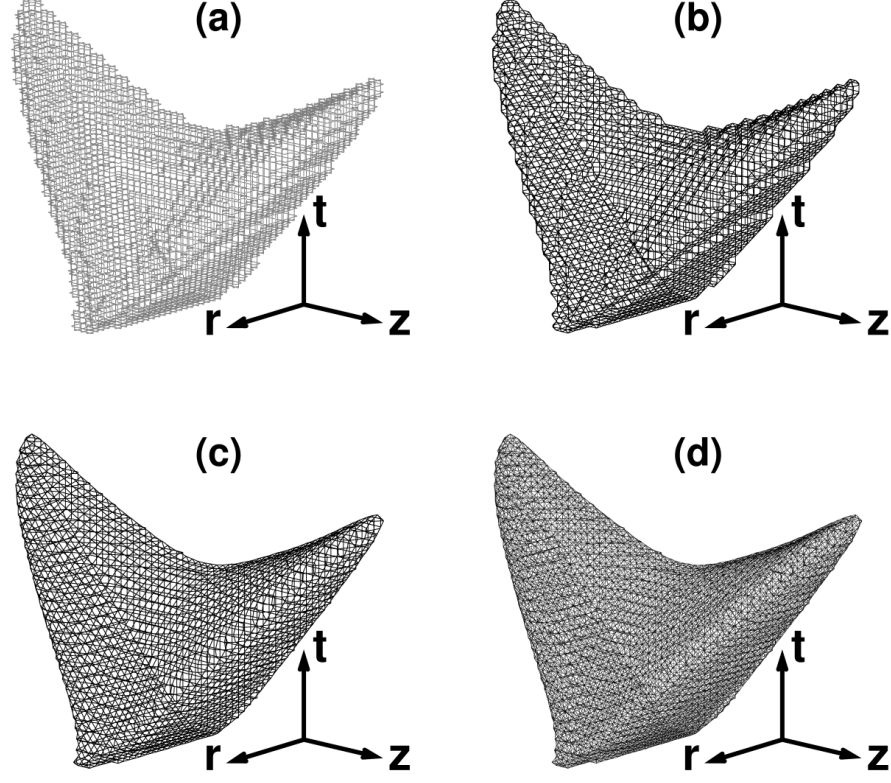


FIG. 32: Various intermediate results of the VESTA processing: (a) wire frame of voxel face vectors; (b) VESTA wire frame for the selected voxels shown in Fig. 31.d; (c) wire frame after support point displacement; (d) wire frame after the breakup of the N -cycles into 3-cycles (see text).

of isocontours from the $r - t$ planes at fixed z values as shown in Fig. 31.b.

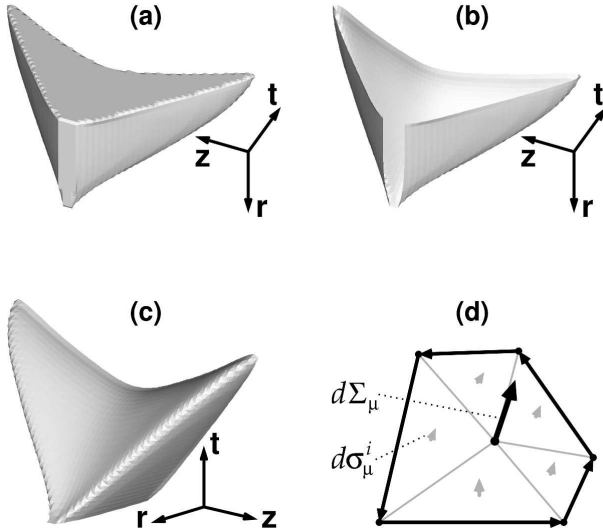


FIG. 33: (a) VESTA surface rendering for the wire frame shown in Fig. 32.d; (b) as in (a), but after the removal of unphysical triangles; (c) as in (b), but rotated VESTA freezeout hypersurface with $T_f = 139 \text{ MeV}$ for the 2+1D hydrodynamic simulation data; (d) decomposed VESTA 5-cycle with its corresponding normal vectors (see text).

In order to avoid any correspondence problems within 3D, the 2+1D relativistic hydrodynamic simulation code HYLANDER-C starts nowadays from the consideration of the full 3D stack of voxels, which each represent a temperature that is equal to or higher than T_f (*cf.*, Fig. 31.d). The application of VESTA (*cf.*, Fig. 32), while using average points for the breakup of the N -cycles into 3-cycles as explained above, yields the isosurface for the isovalue $T_f = 139 \text{ MeV}$, which is depicted in Figs 33.b and 33.c. During the process of FOHS construction, all unphysical triangles have been discarded, i.e., those, which have (at least) one corner with a radial and/or a temporal value that is smaller than zero.

Note that the union of all isocontour edges, which are shown in Figs 31.a – 31.c, is also given through the set of triangular VESTA surface mesh edges as shown in Fig. 32.c (without the unphysical ones) and vice versa. The gray shading of each triangle in Figs 33.a – 33.c is determined from the knowledge of the normal vector of each of the triangles of the FOHS. Benchmark comparisons between the various VESTA and MCA codes are provided in Table 4. Although the extended MCA is here the fastest, it also creates the most redundant information, i.e., 19,218 support points instead of only 3,326.

Technique	original VESTA			marching VESTA			extended MCA
Connectivity	Disconnect	Connect	Mixed	Disconnect	Connect	Mixed	Disconnect
3-Cycles	726	726	726	726	726	726	726
4-Cycles	1,850	1,850	1,850	1,850	1,850	1,850	1,850
5-Cycles	532	532	532	532	532	532	532
6-Cycles	96	96	96	96	96	96	96
7-Cycles	0	0	0	0	0	0	0
8-Cycles	N/A	N/A	0	N/A	N/A	0	N/A
9-Cycles	N/A	N/A	0	N/A	N/A	0	N/A
12-Cycles	N/A	N/A	0	N/A	N/A	0	N/A
Cycle Sum	3,204	3,204	3,204	3,204	3,204	3,204	3,204
L: Points	3,326	3,326	3,326	12,814	12,814	12,814	19,218
L: Triangles	6,406	6,406	6,406	6,406	6,406	6,406	6,406
L: Time [s]	0.124(2)	0.124(1)	0.123(1)	0.108(1)	0.105(1)	0.108(1)	0.096(1)
H: Points	5,804	5,804	5,804	15,292	15,292	15,292	N/A
H: Triangles	11,362	11,362	11,362	11,362	11,362	11,362	N/A
H: Time [s]	0.122(1)	0.124(1)	0.123(3)	0.109(2)	0.109(1)	0.109(1)	N/A

TABLE IV: Relativistic hydrodynamic simulation data benchmark: 33 images with dimensions $241 \times 81 = 19,521$ pixels each; isovalue equals to 139; number of active voxels: 8,000, i.e., 1.242%.

Since the number of normal vectors can be quite large for a generated FOHS, it may be desirable to reduce this number for the speed up of subsequent, further calculations. In Fig. 33.d, a single decomposed VESTA 5-cycle is shown. To each triangle, its normal vector (gray), $d\sigma_\mu^i$ ($i = 1, \dots, 5$), has been attached to the average (or “center of mass”) point of each the three triangle corner points. Note that the orientations of these normal vectors have been inherited from the initial orientation of the VESTA 5-cycle.

In order to prevent further averaging of other field quantities, which are potentially present within the initial 3D voxel data set, and because we intend to speed up subsequent calculations, it is adequate to construct the total normal vector, $d\Sigma_\mu$, as the sum of the triangle normals, $d\sigma_\mu^i$

$$d\Sigma_\mu = \sum_{i=1}^N d\sigma_\mu^i \quad (1)$$

where $d\Sigma_\mu$ is associated with the average (or “center of mass”) point of each support point of the VESTA N -cycle under consideration. Note that in the case of VESTA 3-cycles, $d\Sigma_\mu \equiv d\sigma_\mu^1$. In this case, $d\Sigma_\mu$ is associated with the center of mass point of the corresponding single triangle. For further considerations of normal vector construction, we would like to refer the reader to Ref. [31].

IV. SUMMARY

In summary, all the details that are necessary for a successful implementation of the volume-enclosing surface extraction algorithm, which has been named VESTA, have been explained here for the very first time. VESTA surfaces are always perfect in the sense, that they are non-degenerate, i.e., they always fully enclose a volume that is larger than zero, and they never self-intersect and/or overlap each other (i.e., prior to a possible move of the initial boundary face centers within the bounds of the corresponding range vectors).

We would like to stress that holes can never occur within a VESTA surface. Hence, VESTA surfaces do not require any kind of repair since potential ambiguities are correctly resolved. Furthermore, the information of the interior/exterior of the enclosed shapes is propagated also correctly at all times. In particular, the VESTA surfaces may be viewed as the 3D analog of the DICONEX contours in 2D. Note that VESTA could be extended to adaptive and/or unstructured 3D grids as well. Furthermore, it is fully compatible with its 4D counterpart, STEVE [34], which is capable of the processing of time-varying voxel data. In fact, the ideas which have been presented in this paper can be applied to n -dimensional spaces [35].

The fact that VESTA surface cycles are confined to $2 \times 2 \times 2$ voxel neighborhoods is a result, but not a prerequisite as it is the case in Refs. [4, 9, 10]. Unlike the mesh generators presented in Ref.s [4, 10], VESTA is not template-based. Instead, VESTA uses the single building block, which is shown in Fig. 2.b. VESTA will find all of the required surface segments, which are determined

with a fast and 100% robust construction technique.

We have demonstrated the usage of VESTA for several rather diverse applications, namely for the creation of isosurfaces based on 3D image data in the fields computer tomography and x-ray microtomography, respectively, as well as for the creation of a freezeout hypersurface from a set of 3D numerical simulation data in the field of relativistic heavy-ion physics. Various benchmarks have shown that VESTA can easily compete with the Marching Cubes algorithm, e.g., as far as computing speed is concerned. In addition, VESTA can produce six

different types of surface outputs.

V. ACKNOWLEDGEMENTS

This work has been supported by the Department of Energy under contract W-7405-ENG-36. I am indebted to Dr. Dan Strottman for the extended support after my return to Germany.

-
- [1] J. C. Russ, *The Image Processing Handbook*, CRC Press, 1998.
 - [2] G. Lohmann, *Volumetric Image Analysis*, John Wiley & Sons, 1998.
 - [3] J. D. Foley, A. van Dam, S. K. Feiner, J. F. Hughes, R. L. Phillips, *Introduction to Computer Graphics*, Addison-Wesley Longman, Amsterdam, 1993.
 - [4] W. E. Lorenzen and H. E. Cline, "Marching Cubes: A High Resolution 3D Surface Construction Algorithm," *Comput. Graph.* **21** (1987), pp. 163 – 169.
 - [5] Nvidia, K. Akeley, H. Nguyen, *GPU Gems 3: Programming Techniques for High-Performance Graphics and General-Purpose Computation*, Addison-Wesley Professional, 2007.
 - [6] A. Doi, A. Koide, "An Efficient Method of Triangulating Equivalued Surfaces by using Tetrahedral Cells," *IEICE Transactions Communication, Elec. Info. Syst.* **E74**(1), pp. 214 – 224, 1991.
 - [7] A. Guezic, R. Hummel, "Exploiting Triangulated Surface Extraction using Tetrahedral Decomposition," *IEEE Transactions on Visualization and Computer Graphics*, **1**(4), pp. 328 – 342, 1995.
 - [8] J.-P. Thirion, A. Gourdon, "The 3D Marching Lines Algorithm," *Graphical Models and Image Processing*, **58**, pp. 503 – 509, 1996.
 - [9] J. Bloomenthal, "An Implicit Surface Polygonizer," in *Graphics Gems IV*, ed. P. S. Heckbert, AP Professional, Boston, 1994, pp. 324 – 349.
 - [10] C.-C. Ho, F.-C. Wu, B.-Y. Chen, Y.-Y. Chuang, M. Ouhyoung, "Cubical Marching Squares: Adaptive Feature Preserving Surface Extraction from Volume Data," *Computer Graphics Forum* **24** (2005), pp. 537 – 545, doi: 10.1111/j.1467-8659.2005.00879.x.
 - [11] S. Hill, J. C. Roberts, "Surface Models and the Resolution of N-Dimensional Cell Ambiguity," in *Graphics Gems V*, ed. A. W. Peath, Academic Press, 1995, pp. 98 – 106.
 - [12] E. Chernyaev, "Marching cubes 33: Construction of Topologically Correct Isosurfaces," *CERN Report*, CN/95-17, 1995.
 - [13] T. Lewiner, H. Lopes, A. Viera, G. Tavares, "Efficient implementation of marching cubes' cases with topological guarantees," *Journal of Graphics Tools* **8**(2), pp. 1 – 15, ACM Press, 2003.
 - [14] T. S. Newman, and H. Yi, "A Survey of the Marching Cubes Algorithm," *Computers & Graphics* **30** (2006), pp. 854 – 879, doi: 10.1016/j.cag.2006.07.021.
 - [15] B. R. Schlei, "VESTA - Surface Extraction," Theoretical Division - Self Assessment, Special Feature, a portion of LA-UR-03-3000, Los Alamos (2003) 37.
 - [16] B. R. Schlei, "VESTA - Volume-Enclosing Surface Extraction Algorithm, Version 1.0," Los Alamos Computer Code LA-CC-04-057, Los Alamos National Laboratory.
 - [17] B. R. Schlei, "A New Computational Framework for 2D Shape-Enclosing Contours," *Image and Vision Computing* **27** (2009) 637, doi: 10.1016/j.imavis.2008.06.014.
 - [18] P. Bourke, "Polygonising a scalar field," May 1994; for more detail, cf., <http://paulbourke.net/geometry/polygonise/>
 - [19] W. Boehm, M. Paluszny, H. Prautzsch, *Bezier and B-Spline Techniques*, Springer, Berlin, 2002.
 - [20] D. Salomon, *Curves and Surfaces for Computer Graphics*, Springer, Berlin, 2005.
 - [21] G. T. Herman, *Fundamentals of Computerized Tomography: Image reconstruction from projection*, Springer, 2009.
 - [22] S. R. Stock, *MicroComputed Tomography: Methodology and Applications*, CRC Press, 2008.
 - [23] J. Bolz, U. Ornik, R. M. Weiner, "Relativistic Hydrodynamics of Partially Stopped Baryonic Matter," *Phys. Rev.* **C46** (1992) 2047 – 2056.
 - [24] J. Bolz, U. Ornik, M. Plümer, B. R. Schlei, R. M. Weiner, "Resonance Decays and Partial Coherence in Bose-Einstein Correlations," *Phys. Rev.* **D47** (1993) 3860 – 3870.
 - [25] T. S. Yoo, CT scan of the Stanford terra-cotta bunny; for more detail, cf., <http://graphics.stanford.edu/data/voldata/>
 - [26] Courtesy of G. T. Seidler, Physics Department, University of Washington, Seattle.
 - [27] B. L. Friman, C. Höhne, J. Knoll, S. Leupold, J. Randrup, R. Rapp, P. Senger, editors, *The CBM Physics Book: Compressed Baryonic Matter in Laboratory Experiments* (Lecture Notes in Physics), Berlin: Springer, 2011.
 - [28] R. B. Clare, D. Strottman, "Relativistic Hydrodynamics and Heavy Ion Collisions," *Phys. Rep.* **141** (1986) 177 – 280.
 - [29] L. P. Csernai, *Introduction to Relativistic Heavy Ion Collisions*, John Wiley & Sons, 1994.
 - [30] F. Cooper, G. Frye, E. Schonberg, "Landau's Hydrodynamic Model of Particle Production and Electron-Positron Annihilation into Hadrons," *Phys. Rev.* **D11** (1975) 192 – 213.

- [31] Yun Cheng, L. P. Csernai, V. K. Magas, B. R. Schlei, and D. Strottman, “Matching Stages of Heavy-Ion Collision Models,” *Phys. Rev.* **C81**, 064910 (2010), doi: 10.1103/PhysRevC.81.064910.
- [32] B. R. Schlei, “Extracting the Equation of State of Nuclear Matter through Hydrodynamical Analysis,” *Heavy Ion Phys.* **5**, 403 – 415 (1997).
- [33] B. R. Schlei, D. Strottman, “Predictions for $\sqrt{s} = 200\text{A}GeV$ Au+Au Collisions from Relativistic Hydrodynamics,” *Phys. Rev.* **C59**, R9 – R12 (1999).
- [34] B. R. Schlei, “Hyper-Surface Extraction in Four Dimensions,” Theoretical Division - Self Assessment, Special Feature, a portion of LA-UR-04-2143, Los Alamos (2004) 168.
- [35] B. R. Schlei, “Verfahren zur Hyperflächenkonstruktion in n Dimensionen,” German Patent Applications No. 10 2011 050 721.3 and No. 10 2011 051 203.9; submitted by GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstr. 1, 64291 Darmstadt, Germany, on May 30, 2011, and June 20, 2011, respectively.
- [36] More complicated types of interpolations than just simple linear interpolations are possible. E.g., one could use (higher-dimensional) B-Splines[19, 20] instead, etc.